PROBLEMS

6.1 Employ fixed-point iteration to locate the root of

$$f(x) = 2\sin(\sqrt{x}) - x$$

Use an initial guess of $x_0 = 0.5$ and iterate until $\varepsilon_a \le 0.01\%$.

6.2 Use **(a)** fixed-point iteration and **(b)** the Newton-Raphson method to determine a root of $f(x) = -x^2 + 1.8x + 2.5$ using $x_0 = 5$. Perform the computation until ε_a is less than $\varepsilon_s = 0.05\%$. Also check your final answer.

6.3 Determine the highest real root of $f(x) = 0.95x^3 - 5.9x^2 + 10.9x - 6$:

(a) Graphically.

(b) Using the Newton-Raphson method (three iterations, $x_i = 3.5$).

(c) Using the secant method (three iterations, $x_{i-1} = 2.5$ and $x_i = 3.5$).

(d) Using the modified secant method (three iterations, $x_i = 3.5$, $\delta = 0.01$).

(e) Determine all the roots with MATLAB.

6.4 Determine the lowest positive root of $f(x) = 8\sin(x)e^{-x} - 1$:

(a) Graphically.

(b) Using the Newton-Raphson method (three iterations, $x_i = 0.3$).

(c) Using the secant method (three iterations, $x_{i-1} = 0.5$ and $x_i = 0.4$.

(d) Using the modified secant method (five iterations, $x_i = 0.3$, $\delta = 0.01$).

6.5 Use **(a)** the Newton-Raphson method and **(b)** the modified secant method ($\delta = 0.05$) to determine a root of $f(x) = x^5 - 16.05x^4 + 88.75x^3 - 192.0375x^2 + 116.35x + 31.6875$ using an initial guess of $x = 0.5825$ and $\varepsilon_s = 0.01\%$. Explain your results.

6.6 Develop an M-file for the secant method. Along with the two initial guesses, pass the function as an argument. Test it by solving Prob. 6.3.

6.7 Develop an M-file for the modified secant method. Along with the initial guess and the perturbation fraction, pass the function as an argument. Test it by solving Prob. 6.3.

6.8 Differentiate Eq. (E6.4.1) to get Eq. (E6.4.2).

6.9 Employ the Newton-Raphson method to determine a real root for $f(x) = -1 + 6x - 4x^2 + 0.5x^3$, using an initial guess of **(a)** 4.5, and **(b)** 4.43. Discuss and use graphical and analytical methods to explain any peculiarities in your results.

6.10 The "divide and average" method, an old-time method for approximating the square root of any positive number a, can be formulated as

$$x_{i+1} = \frac{x_i + a/x_i}{2}$$

Prove that this formula is based on the Newton-Raphson algorithm.

6.11 **(a)** Apply the Newton-Raphson method to the function $f(x) = \tanh(x^2 - 9)$ to evaluate its known real root at $x = 3$. Use an initial guess of $x_0 = 3.2$ and take a minimum of three iterations. **(b)** Did the method exhibit convergence onto its real root? Sketch the plot with the results for each iteration labeled.

6.12 The polynomial $f(x) = 0.0074x^4 - 0.284x^3 + 3.355x^2 - 12.183x + 5$ has a real root between 15 and 20. Apply the Newton-Raphson method to this function using an initial guess of $x_0 = 16.15$. Explain your results.

6.13 In a chemical engineering process, water vapor (H_2O) is heated to sufficiently high temperatures that a significant portion of the water dissociates, or splits apart, to form oxygen (O_2) and hydrogen (H_2):

$$H_2O \rightleftarrows H_2 + \tfrac{1}{2}O_2$$

If it is assumed that this is the only reaction involved, the mole fraction x of H_2O that dissociates can be represented by

$$K = \frac{x}{1-x}\sqrt{\frac{2p_t}{2+x}} \qquad \text{(P6.13.1)}$$

where K is the reaction's equilibrium constant and p_t is the total pressure of the mixture. If $p_t = 3.5$ atm and $K = 0.04$, determine the value of x that satisfies Eq. (P6.13.1).

6.14 The Redlich-Kwong equation of state is given by

$$p = \frac{RT}{v-b} - \frac{a}{v(v+b)\sqrt{T}}$$

where R = the universal gas constant [= 0.518 kJ/(kg K)], T = absolute temperature (K), p = absolute pressure (kPa), and v = the volume of a kg of gas (m^3/kg). The parameters a and b are calculated by

$$a = 0.427\frac{R^2T_c^{2.5}}{p_c} \qquad b = 0.0866R\frac{T_c}{p_c}$$

where $p_c = 4580$ kPa and $T_c = 191$ K. As a chemical engineer, you are asked to determine the amount of methane fuel that can be held in a 3-m^3 tank at a temperature of −50 °C with a pressure of 65,000 kPa. Use a root locating method of your choice to calculate v and then determine the mass of methane contained in the tank.

6.15 The volume of liquid V in a hollow horizontal cylinder of radius r and length L is related to the depth of the liquid h by

$$V = \left[r^2\cos^{-1}\left(\frac{r-h}{r}\right) - (r-h)\sqrt{2rh - h^2}\right]L$$

Determine h given $r = 2$ m, $L = 5$ m^3, and $V = 8.5$ m^3.

FI

6.16 A catenary cable i
points not in the sam
Fig. P6.16*a*, it is subje
weight. Thus, its weigh
length along the cable *w*
section *AB* is depicted in
the tension forces at the e
cal force balances, the fol
of the cable can be derive

$$\frac{d^2y}{dx^2} = \frac{w}{T_A}\sqrt{1 + \left(\frac{dy}{dx}\right.}$$

Calculus can be employ
height of the cable *y* as a

$$y = \frac{T_A}{w}\cosh\left(\frac{w}{T_A}x\right)$$

(a) Use a numerical me
parameter T_A given
and $y_0 = 6$, such that
$x = 50$.

(b) Develop a plot of y v

6.17 An oscillating curre
by $I = 9e^{-t}\cos(2\pi t)$, w
values of t such that I =

6.18 Figure P6.18 shows
tor, and a capacitor in pa
to express the impedance

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \right.}$$

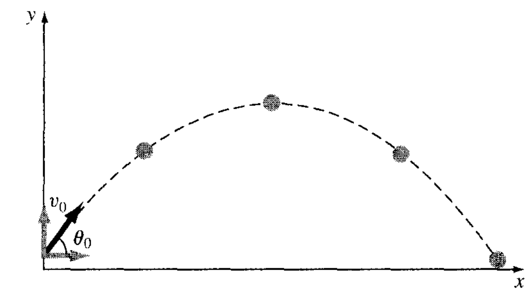

FIGURE P6.20

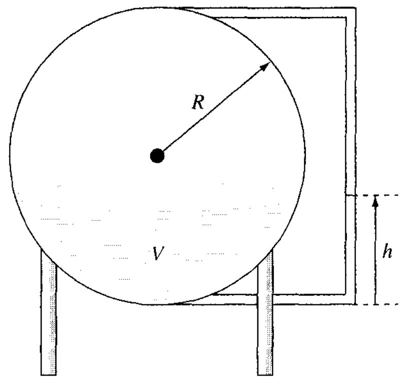

FIGURE P6.21

released a distance h above a nonlinear spring. The resistance force F of the spring is given by

$$F = -(k_1 d + k_2 d^{3/2})$$

Conservation of energy can be used to show that

$$0 = \frac{2k_2 d^{5/2}}{5} + \frac{1}{2}k_1 d^2 - mgd - mgh$$

Solve for d, given the following parameter values: $k_1 = 50{,}000\ \text{g/s}^2$, $k_2 = 40\ \text{g/(s}^2\ \text{m}^5)$, $m = 90$ g, $g = 9.81\ \text{m/s}^2$, and $h = 0.45$ m.

6.20 Aerospace engineers sometimes compute the trajectories of projectiles such as rockets. A related problem deals with the trajectory of a thrown ball. The trajectory of a ball thrown by a right fielder is defined by the (x, y) coordinates as displayed in Fig. P6.20. The trajectory can be modeled as

$$y = (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2 + y_0$$

Find the appropriate initial angle θ_0, if $v_0 = 20$ m/s, and the distance to the catcher is 35 m. Note that the throw leaves the right fielder's hand at an elevation of 2 m and the catcher receives it at 1 m.

6.21 You are designing a spherical tank (Fig. P6.21) to hold water for a small village in a developing country. The volume of liquid it can hold can be computed as

$$V = \pi h^2\frac{[3R - h]}{3}$$

where V = volume [m^3], h = depth of water in tank [m], and R = the tank radius [m].

If $R = 3$ m, what depth must the tank be filled to so that it holds 30 m^3? Use three iterations of the most efficient numerical method possible to determine your answer. Determine the approximate relative error after each iteration. Also, provide justification for your choice of method. Extra information: **(a)** For bracketing methods, initial guesses of 0 and R will bracket a single root for this example. **(b)** For open methods, an initial guess of R will always converge.

6.22 Perform the identical MATLAB operations as those in Example 6.7 to manipulate and find all the roots of the polynomial

$$f_5(x) = (x + 2)(x + 5)(x - 1)(x - 4)(x - 7)$$

6.23 In control systems analysis, transfer functions are developed that mathematically relate the dynamics of a system's input to its output. A transfer function for a robotic positioning system is given by

$$G(s) = \frac{C(s)}{N(s)} = \frac{s^3 + 12.5s^2 + 50.5s + 66}{s^4 + 19s^3 + 122s^2 + 296s + 192}$$

where $G(s)$ = system gain, $C(s)$ = system output, $N(s)$ = system input, and s = Laplace transform complex frequency. Use MATLAB to find the roots of the numerator and denominator and factor these into the form

$$G(s) = \frac{(s + a_1)(s + a_2)(s + a_3)}{(s + b_1)(s + b_2)(s + b_3)(s + b_4)}$$

where a_i and b_i = the roots of the numerator and denominator, respectively.

6.24 The Manning equation can be written for a rectangular open channel as

$$Q = \frac{\sqrt{S}(BH)^{5/3}}{n(B + 2H)^{2/3}}$$

where Q = flow (m^3/s), S = slope (m/m), H = depth (m), and n = the Manning roughness coefficient. Develop a

fixed-point iteration sch
given $Q = 5$, $S = 0.0002$
your scheme converges fo
equal to zero.
6.25 See if you can devel
the friction factor based
described in Sec. 6.6. You
result for Reynolds numbe
ε/D ranging from 0.0000
6.26 Use the Newton-Ra

$f(x) = e^{-0.5x}(4 - x$

Employ initial guesses of (
results.
6.27 Given

$f(x) = -2x^6 - 1.5x$

Use a root location technic
this function. Perform itera
tive error falls below 5%.

fixed-point iteration scheme to solve this equation for H given $Q = 5$, $S = 0.0002$, $B = 20$, and $n = 0.03$. Prove that your scheme converges for all initial guesses greater than or equal to zero.

6.25 See if you can develop a foolproof function to compute the friction factor based on the Colebrook equation as described in Sec. 6.6. Your function should return a precise result for Reynolds number ranging from 4000 to 10^7 and for ε/D ranging from 0.00001 to 0.05.

6.26 Use the Newton-Raphson method to find the root of

$$f(x) = e^{-0.5x}(4 - x) - 2$$

Employ initial guesses of **(a)** 2, **(b)** 6, and **(c)** 8. Explain your results.

6.27 Given

$$f(x) = -2x^6 - 1.5x^4 + 10x + 2$$

Use a root location technique to determine the maximum of this function. Perform iterations until the approximate relative error falls below 5%. If you use a bracketing method, use initial guesses of $x_l = 0$ and $x_u = 1$. If you use the Newton-Raphson or the modified secant method, use an initial guess of $x_i = 1$. If you use the secant method, use initial guesses of $x_{i-1} = 0$ and $x_i = 1$. Assuming that convergence is not an issue, choose the technique that is best suited to this problem. Justify your choice.

6.28 You must determine the root of the following easily differentiable function:

$$e^{0.5x} = 5 - 5x$$

Pick the best numerical technique, justify your choice, and then use that technique to determine the root. Note that it is known that for positive initial guesses, all techniques except fixed-point iteration will eventually converge. Perform iterations until the approximate relative error falls below 2%. If you use a bracketing method, use initial guesses of $x_l = 0$ and $x_u = 2$. If you use the Newton-Raphson or the modified secant method, use an initial guess of $x_i = 0.7$. If you use the secant method, use initial guesses of $x_{i-1} = 0$ and $x_i = 2$.

15.16 You measure the voltage drop V across a resistor for a number of different values of current i. The results are

i	0.25	0.75	1.25	1.5	2.0
V	−0.45	−0.6	0.70	1.88	6.0

Use first- through fourth-order polynomial interpolation to estimate the voltage drop for $i = 1.15$. Interpret your results.

15.17 The current in a wire is measured with great precision as a function of time:

t	0	0.1250	0.2500	0.3750	0.5000
i	0	6.24	7.75	4.85	0.0000

Determine i at $t = 0.23$.

15.18 The acceleration due to gravity at an altitude y above the surface of the earth is given by

y, m	0	30,000	60,000	90,000	120,000
g, m/s^2	9.8100	9.7487	9.6879	9.6278	9.5682

Compute g at $y = 55{,}000$ m.

TABLE P15.19 Temperatures (°C) at various points on a square heated plate.

	$x = 0$	$x = 2$	$x = 4$	$x = 6$	$x = 8$
$y = 0$	100.00	90.00	80.00	70.00	60.00
$y = 2$	85.00	64.49	53.50	48.15	50.00
$y = 4$	70.00	48.90	38.43	35.03	40.00
$y = 6$	55.00	38.78	30.39	27.07	30.00
$y = 8$	40.00	35.00	30.00	25.00	20.00

15.19 Temperatures are measured at various points on a heated plate (Table P15.19). Estimate the temperature at **(a)** $x = 4$, $y = 3.2$, and **(b)** $x = 4.3$, $y = 2.7$.

15.20 Use the portion of the given steam table for superheated H_2O at 200 MPa to **(a)** find the corresponding entropy s for a specific volume v of 0.108 m^3/kg with linear interpolation, **(b)** find the same corresponding entropy using quadratic interpolation, and **(c)** find the volume corresponding to an entropy of 6.6 using inverse interpolation.

v (m^3/kg)	0.10377	0.11144	0.12540
s (kJ/kg·K)	6.4147	6.5453	6.7664