

where  $w(z)$  = width of the dam face (m) at elevation  $z$  (Fig. P17.9b). The line of action can also be obtained by evaluating

$$d = \frac{\int_0^D \rho g z w(z)(D - z) dz}{\int_0^D \rho g w(z)(D - z) dz}$$

Use Simpson's rule to compute  $f_i$  and  $d$ .

**17.10** The force on a sailboat mast can be represented by the following function:

$$f(z) = 200 \left( \frac{z}{7+z} \right) e^{-2.5z/H}$$

where  $z$  = the elevation above the deck and  $H$  = the height of the mast. The total force  $F$  exerted on the mast can be determined by integrating this function over the height of the mast:

$$F = \int_0^H f(z) dz$$

The line of action can also be determined by integration:

$$d = \frac{\int_0^H z f(z) dz}{\int_0^H f(z) dz}$$

(a) Use the composite trapezoidal rule to compute  $F$  and  $d$  for the case where  $H = 30$  ( $n = 6$ ).

(b) Repeat (a), but use the composite Simpson's 1/3 rule.

**17.11** A wind force distributed against the side of a skyscraper is measured as

Height $l$ , m	0	30	60	90	120
Force, $F(l)$ , N/m	0	340	1200	1600	2700

Height $l$ , m	150	180	210	240
Force, $F(l)$ , N/m	3100	3200	3500	3800

Compute the net force and the line of action due to this distributed wind.

**17.12** An 11-m beam is subjected to a load, and the shear force follows the equation

$$V(x) = 5 + 0.25x^2$$

where  $V$  is the shear force, and  $x$  is length in distance along the beam. We know that  $V = dM/dx$ , and  $M$  is the bending moment. Integration yields the relationship

$$M = M_o + \int_0^x V dx$$

If  $M_o$  is zero and  $x = 11$ , calculate  $M$  using (a) analytical integration, (b) multiple-application trapezoidal rule, and (c) multiple-application Simpson's rules. For (b) and (c) use 1-m increments.

**17.13** The total mass of a variable density rod is given by

$$m = \int_0^L \rho(x) A_c(x) dx$$

where  $m$  = mass,  $\rho(x)$  = density,  $A_c(x)$  = cross-sectional area,  $x$  = distance along the rod and  $L$  = the total length of the rod. The following data has been measured for a 10-m length rod. Determine the mass in grams to the best possible accuracy.

$x$ , m	0	2	3	4	6	8	10
$\rho$ , g/cm <sup>3</sup>	4.00	3.95	3.89	3.80	3.60	3.41	3.30
$A_c$ , cm <sup>2</sup>	100	103	106	110	120	133	150

**17.14** A transportation engineering study requires that you determine the number of cars that pass through an intersection traveling during morning rush hour. You stand at the side of the road and count the number of cars that pass every 4 minutes at several times as tabulated below. Use the best numerical method to determine (a) the total number of cars that pass between 7:30 and 9:15, and (b) the rate of cars going through the intersection per minute. (Hint: Be careful with units.)

Time (hr)	7:30	7:45	8:00	8:15	8:45	9:15
Rate (cars per 4 min)	18	24	14	24	21	9

**17.15** Determine the average value for the data in Fig. P17.15. Perform the integral needed for the average in the order shown by the following equation:

$$I = \int_{x_0}^{x_n} \left[ \int_{y_0}^{y_m} f(x, y) dy \right] dx$$

**17.16** Integration provides a means to compute how much mass enters or leaves a reactor over a specified time period, as in

$$M = \int_{t_1}^{t_2} Q_c dt$$

where  $t_1$  and  $t_2$  = the initial and final times, respectively. This formula makes intuitive sense if you recall the analogy between integration and summation. Thus, the integral represents the summation of the product of flow times concentration to give the total mass entering or leaving from  $t_1$  to  $t_2$ .

Use numerical data listed below

$t$ , min  
 $Q$ , m<sup>3</sup>/min  
 $c$ , mg/m<sup>3</sup>

**17.17** The c  
puted as

$$A_c = \int_0^B$$

where  $B = t$   
and  $y = \text{dist}$   
average flow

$$Q = \int_0^h$$

where  $U =$   
a numerical  
data:

$y$ , m  
 $H$ , m  
 $U$ , m/s

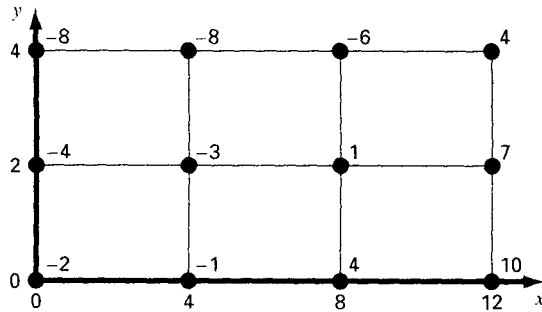


FIGURE P17.15

Use numerical integration to evaluate this equation for the data listed below:

$t, \text{ min}$	0	10	20	30	35	40	45	50
$Q, \text{ m}^3/\text{min}$	4	4.8	5.2	5.0	4.6	4.3	4.3	5.0
$c, \text{ mg/m}^3$	10	35	55	52	40	37	32	34

17.17 The cross-sectional area of a channel can be computed as

$$A_c = \int_0^B H(y) dy$$

where  $B$  = the total channel width (m),  $H$  = the depth (m), and  $y$  = distance from the bank (m). In a similar fashion, the average flow  $Q$  ( $\text{m}^3/\text{s}$ ) can be computed as

$$Q = \int_0^B U(y)H(y) dy$$

where  $U$  = water velocity (m/s). Use these relationships and a numerical method to determine  $A_c$  and  $Q$  for the following data:

$y, \text{ m}$	0	2	4	5	6	9
$H, \text{ m}$	0.5	1.3	1.25	1.7	1	0.25
$U, \text{ m/s}$	0.03	0.06	0.05	0.12	0.11	0.02

17.18 The average concentration of a substance  $\bar{c}$  ( $\text{g/m}^3$ ) in a lake where the area  $A_s$  ( $\text{m}^2$ ) varies with depth  $z$  (m) can be computed by integration:

$$\bar{c} = \frac{\int_0^Z c(z)A_s(z) dz}{\int_0^Z A_s(z) dz}$$

where  $Z$  = the total depth (m). Determine the average concentration based on the following data:

$z, \text{ m}$	0	4	8	12	16
$A, 10^6 \text{ m}^2$	9.8175	5.1051	1.9635	0.3927	0.0000
$c, \text{ g/m}^3$	10.2	8.5	7.4	5.2	4.1

17.19 As was done in Section 17.9, determine the work performed if a constant force of 1 N applied at an angle  $\theta$  results in the following displacements. Use the `cumtrapz` function to determine the cumulative work and plot the result versus  $\theta$ .

$x, \text{ m}$	0	1	2.7	3.8	3.7	3	1.4
$\theta, \text{ rad}$	0	30	60	90	120	150	180