SECTION 2-10

Leaching

PROBLEM 10.1

0.4 kg/s of dry sea-shore sand, containing 1 per cent by mass of salt, is to be washed with 0.4 kg/s of fresh water running countercurrently to the sand through two classifiers in series. It may be assumed that perfect mixing of the sand and water occurs in each classifier and that the sand discharged from each classifier contains one part of water for every two of sand by mass. If the washed sand is dried in a kiln dryer, what percentage of salt will it retain? What wash rate would be required in a single classifier in order to wash the sand to the same extent?

Solution

The problem involves a mass balance around the two stages. If x kg/s salt is in the underflow discharge from stage 1, then:

$$\text{salt in feed to stage } 2 = (0.4 \times 1)/100 = 0.004 \text{ kg/s.}$$

The sand passes through each stage and hence the sand in the underflow from stage $1 = 0.4$ kg/s, which, assuming constant underflow, is associated with $(0.4/2) = 0.2$ kg/s water. Similarly, 0.2 kg/s water enters stage 1 in the underflow and 0.4 kg/s enters in the overflow. Making a water balance around stage 1, the water in the overflow discharge = 0.4 kg/s.

In the underflow discharge from stage 1, x kg/s salt is associated with 0.2 kg/s water, and hence the salt associated with the 0.4 kg/s water in the overflow discharge $= (x \times 0.4)/0.2 = 2x$ kg/s. This assumes that the overflow and underflow solutions have the same concentration.

In stage 2, 0.4 kg/s water enters in the overflow and 0.2 kg/s leaves in the underflow.

Thus: $$\text{water in overflow from stage } 2 = (0.4 - 0.2) = 0.2 \text{ kg/s.}$$

The salt entering is 0.004 kg/s in the underflow and $2x$ in the overflow — a total of $(0.004 + 2x)$ kg/s. The exit underflow and overflow concentrations must be the same, and hence the salt associated with 0.2 kg/s water in each stream is:

$$(0.004 + 2x)/2 = (0.002 + x) \text{ kg/s}$$

Making an overall salt balance:

$$0.004 = x + (0.002 + x) \quad \text{and} \quad x = 0.001 \text{ kg/s}$$

This is associated with 0.4 kg/s sand and hence:

$$\text{salt in dried sand} = (0.001 \times 100)/(0.4 + 0.001) = \underline{\underline{0.249 \text{ per cent}}}$$

The same result may be obtained by applying equation 10.16 over the washing stage:

$$S_{n+1}/S_1 = (R - 1)/(R^{n+1} - 1) \qquad \text{(equation 10.16)}$$

In this case: $R = (0.4/0.2) = 2, n = 1, S_2 = x, S_1 = (0.002 + x)$ and:

$$x/(0.002 + x) = (2 - 1)/(2^2 - 1) = 0.33$$

$$x = (0.000667/0.667) = 0.001 \text{ kg/s}$$

and the salt in the sand = $\underline{\underline{0.249 \text{ per cent}}}$ as before.

Considering a *single stage*:

If y kg/s is the overflow feed of water then, since 0.2 kg/s water leaves in the underflow, the water in the overflow discharge = $(y - 0.2)$ kg/s. With a feed of 0.004 kg/s salt and 0.001 kg/s salt in the underflow discharge, the salt in the overflow discharge = 0.003 kg/s. The ratio (salt/solution) must be the same in both discharge streams or:

$$(0.001)/(0.20 + 0.001) = 0.003/(0.003 + y - 0.2) \quad \text{and} \quad \underline{\underline{y = 0.8 \text{ kg/s}}}$$

PROBLEM 10.2

Caustic soda is manufactured by the lime-soda process. A solution of sodium carbonate in water containing 0.25 kg/s Na_2CO_3 is treated with the theoretical requirement of lime and, after the reaction is complete, the $CaCO_3$ sludge, containing by mass 1 part of $CaCO_3$ per 9 parts of water is fed continuously to three thickeners in series and is washed countercurrently. Calculate the necessary rate of feed of neutral water to the thickeners, so that the calcium carbonate, on drying, contains only 1 per cent of sodium hydroxide. The solid discharged from each thickener contains one part by mass of calcium carbonate to three of water. The concentrated wash liquid is mixed with the contents of the agitated before being fed to the first thickeners.

Solution

See Volume 2, Example 10.2.

PROBLEM 10.3

How many stages are required for a 98 per cent extraction of a material containing 18 per cent of extractable matter of density 2700 kg/m^3 and which requires 200 volumes of liquid/100 volumes of solid for it to be capable of being pumped to the next stage? The strong solution is to have a concentration of 100 kg/m^3.

Solution

Taking as a basis *100 kg solids fed to the plant*, this contains 18 kg solute and 82 kg inert material. The extraction is 98 per cent and hence $(0.98 \times 18) = 17.64$ kg solute appears in the liquid product, leaving $(18 - 17.64) = 0.36$ kg solute in the washed solid. The concentration of the liquid product is 100 kg/m^3 and hence the volume of the liquid product $= (17.64/100) = 0.1764\ m^3$.

$$\text{Volume of solute in liquid product} = (17.64/2700) = 0.00653\ m^3.$$

$$\text{Volume of solvent in liquid product} = (0.1764 - 0.00653) = 0.1699\ m^3.$$

$$\text{Mass of solvent in liquid product} = 0.1699\rho\ \text{kg}$$

where ρ kg/m^3 is the density of solvent.

$$\text{In the washed solids, total solids} = 82\ \text{kg or}\ (82/2700) = 0.0304\ m^3.$$

$$\text{Volume of solution in the washed solids} = (0.0304 \times 200)/100 = 0.0608\ m^3.$$

$$\text{Volume of solute in solution} = (0.36/2700) = 0.0001\ m^3.$$

$$\text{Volume of solvent in washed solids} = (0.0608 - 0.0001) = 0.0607\ m^3.$$

$$\text{and mass of solvent in washed solids} = 0.0607\rho\ \text{kg}$$

$$\text{Mass of solvent fed to the plant} = (0.0607 + 0.1699)\rho = 0.2306\rho\ \text{kg}$$

The *overall balance* in terms of mass is therefore;

	Inerts	Solute	Solvent
Feed to plant	82	18	—
Wash liquor	—	—	0.2306ρ
Washed solids	82	0.36	0.0607ρ
Liquid product	—	17.64	0.1699ρ

$$\frac{\text{Solvent discharged in the overflow}}{\text{Solvent discharged in the underflow}},\ R = (0.2306\rho/0.0607\rho) = 3.80$$

The overflow product contains 100 kg solute/m^3 solution. This concentration is the same as the underflow from the first thickener and hence the material fed to the washing thickeners contains 82 kg inerts and 0.0608 m^3 solution containing $(100 \times 0.0608) = 6.08$ kg solute.

Thus, in equation 10.16:

$$(3.80 - 1)/(3.80^{n+1} - 1) = (0.36/6.08)$$

or: $3.80^{n+1} = 48.28$ and $n = 1.89$, say 2 washing thickeners.

Thus a total of <u>3 thickeners</u> is required.

PROBLEM 10.4

Soda ash is mixed with lime and the liquor from the second of three thickeners and passed to the first thickener where separation is effected. The quantity of this caustic solution leaving the first thickener is such as to yield 10 Mg of caustic soda per day of 24 hours. The solution contains 95 kg of caustic soda/1000 kg of water, whilst the sludge leaving each of the thickeners consists of one part of solids to one of liquid.

Determine:

(a) the mass of solids in the sludge,
(b) the mass of water admitted to the third thickener and
(c) the percentages of caustic soda in the sludges leaving the respective thickeners.

Solution

Basis: 100 Mg $CaCO_3$ *in the sludge leaving each thickener.*

In order to produce 100 Mg $CaCO_3$, 106 Mg Na_2CO_3 must react giving 80 Mg NaOH according to the equation:

$$Na_2CO_3 + Ca(OH)_2 = 2NaOH + CaCO_3.$$

For the purposes of calculation it is assumed that a mixture of 100 Mg $CaCO_3$ and 80 Mg NaOH is fed to the first thickener and w Mg water is the overflow feed to the third thickener. Assuming that x_1, x_2 and x_3 are the ratios of caustic soda to solution by mass in each thickener then the mass balances are made as follows:

	$CaCO_3$	NaOH	Water
Overall			
Underflow feed	100	80	—
Overflow feed	—	—	w
Underflow product	100	$100x_3$	$100(1 - x_3)$
Overflow product	—	$(80 - 100w_3)$	$w - 100(1 - x_3)$
Thickener 1			
Underflow feed	100	80	—
Overflow feed	—	$100(x_1 - x_3)$	$w + 100(x_3 - x_1)$
Underflow product	100	$100x_1$	$100(1 - x_1)$
Overflow product	—	$80 - 100x_3$	$w - 100(1 - x_3)$
Thickener 2			
Underflow feed	100	$100x_1$	$100(1 - x_1)$
Overflow feed	—	$100(x_2 - x_3)$	$w + 100(x_3 - x_2)$
Underflow product	100	$100x_2$	$100(1 - x_2)$
Overflow product	—	$100(x_1 - x_3)$	$w + 100(x_3 - x_1)$
Thickener 3			
Underflow feed	100	$100x_2$	$100(1 - x_2)$
Overflow feed	—	—	w
Underflow product	100	$100x_3$	$100(1 - x_3$
Overflow product	—	$100(x_2 - x_3)$	$w + 100(x_3 - x_2)$

In the overflow product, 0.095 Mg NaOH is associated with 1 Mg water.

Thus: $$x_1 = 0.095/(1 + 0.095) = 0.0868 \text{ Mg/Mg solution} \quad \text{(i)}$$

Assuming that equilibrium is attained in each thickener, the concentration of NaOH in the overflow product is equal to the concentration of NaOH in the solution in the underflow product.

Thus:
$$x_3 = [100(x_2 - x_3)]/[100(x_2 - x_3) + w - 100(x_2 - x_3)]$$
$$= 100(x_2 - x_3)/w \quad \text{(ii)}$$
$$x_2 = [100(x_1 - x_3)]/[100(x_1 - x_3) + w - 100(x_1 - x_3)]$$
$$= 100(x_1 - x_3)/w \quad \text{(iii)}$$
$$x_3 = (80 - 100x_3)/[80 - 100x_3 + w - 100(1 - x_3)]$$
$$= (80 - 100x_3)/(w - 20) \quad \text{(iv)}$$

Solving equations (i)–(iv) simultaneously, gives:

$$x_3 = 0.0010 \text{ Mg/Mg}, \quad x_2 = 0.0093 \text{ Mg/Mg}, \quad x_1 = 0.0868 \text{ Mg/Mg}$$

and: $w = 940.5$ Mg/100 Mg $CaCO_3$

The overflow product $= w - 100(1 - x_3) = 840.6$ Mg/100 Mg $CaCO_3$.
The actual flow of caustic solution = (10/0.0868) = 115 Mg/day.

Thus: mass of $CaCO_3$ in sludge = (100 × 115)/840.6 = <u>13.7 Mg/day</u>

The mass of water fed to third thickener = 940.5 Mg/100 Mg $CaCO_3$

or: = (940.5 × 13.7)/100 = <u>129 Mg/day</u>

The total mass of sludge leaving each thickener = 200 Mg/100 Mg $CaCO_3$.
The mass of caustic soda in the sludge = $100x_1$ Mg/100 Mg $CaCO_3$ and hence the concentration of caustic in sludge leaving,

thickener 1 = (100 × 0.0868 × 100)/200 = <u>4.34 per cent</u>

thickener 2 = (100 × 0.0093 × 100)/200 = <u>0.47 per cent</u>

thickener 3 = (100 × 0.0010 × 100)/200 = <u>0.05 per cent</u>

PROBLEM 10.5

Seeds, containing 20 per cent by mass of oil, are extracted in a countercurrent plant and 90 per cent of the oil is recovered as a solution containing 50 per cent by mass of oil. If the seeds are extracted with fresh solvent and 1 kg of solution is removed in the underflow in association with every 2 kg of insoluble matter, how many ideal stages are required?

Solution

See Volume 2, Example 10.4.

PROBLEM 10.6

It is desired to recover precipitated chalk from the causticising of soda ash. After decanting the liquor from the precipitators the sludge has the composition 5 per cent $CaCO_3$, 0.1 per cent NaOH and the balance water.

1000 Mg/day of this sludge is fed to two thickeners where it is washed with 200 Mg/day of neutral water. The pulp removed from the bottom of the thickeners contains 4 kg of water/kg of chalk. The pulp from the last thickener is taken to a rotary filter and concentrated to 50 per cent solids and the filtrate is returned to the system as wash water. Calculate the net percentage of $CaCO_3$ in the product after drying.

Solution

Basis: 1000 Mg/day sludge fed to the plant

If x_1 and x_2 are the solute/solvent ratios in thickeners 1 and 2 respectively, then the mass balances are:

	$CaCO_3$	NaOH	Water
Overall			
Underflow feed	50	1	949
Overflow feed	—	—	200
Underflow product	50	$200x_2$	200
Overflow product	—	$(1 - 200x_2)$	949
Thickener 1			
Underflow feed	50	1	949
Overflow feed	—	$200(x_1 - x_2)$	200
Underflow product	50	$200x_1$	200
Overflow product	—	$(1 - 200x_2)$	949
Thickener 2			
Underflow feed	50	$200x_1$	200
Overflow feed	—	—	200
Underflow product	50	$200x_2$	200
Overflow product	—	$200(x_1 - x_2)$	200

Assuming that equilibrium is attained, the solute/solvent ratio will be the same in the overflow and underflow products of each thickener and:

$$x_2 = 200(x_1 - x_2)/200 \quad \text{or} \quad x_2 = 0.5x_1$$

and: $$x_1 = (1 - 200x_2)/949$$

Thus: $$x_1 = 0.000954 \quad \text{and} \quad x_2 = 0.000477$$

The underflow product contains 50 Mg $CaCO_3$, $(200 \times 0.000477) = 0.0954$ Mg NaOH and 200 Mg water. After concentration to 50 per cent solids, the mass of NaOH in solution

$$= (0.0954 \times 50)/200.0954 = 0.0238 \text{ Mg}$$

and the $CaCO_3$ in dried solids

$$= (100 \times 50)/50.0238 = \underline{\underline{99.95 \text{ per cent}}}$$

This approach ignores the fact that filtrate is returned to the second thickener together with wash water. Taking this into account, the calculation is modified as follows.

The underflow product from the second thickener contains:

$$50 \text{ Mg } CaCO_3, 200x_2 \text{ Mg NaOH} \quad \text{and} \quad 200 \text{ Mg water}$$

After filtration, the 50 Mg $CaCO_3$ is associated with 50 Mg solution of the same concentration and hence this contains:

$$50x_2/(1 + x_2) \text{ Mg NaOH} \quad \text{and} \quad 50/(1 + x_2) \text{ Mg water}$$

The remainder is returned with the overflow feed to the second thickener. The filtrate returned contains:

$$200x_2 - 50x_2/(1 + x_2) \text{ Mg NaOH}$$

and: $$200 - 50/(1 + x_2) \text{ Mg water}$$

The balances are now:

	$CaCO_3$	NaOH	Water
Overall			
Underflow feed	50	1	949
Overflow feed	—	$200x_2 - 50x_2/(1 + x_2)$	$400 - 50/(1 + x_2)$
Underflow product	50	$200x_2$	200
Overflow product	—	$1 - 50x_2/(1 + x_2)$	$1149 - 50/(1 + x_2)$
Thickener 1			
Underflow feed	50	1	949
Overflow feed	—	$200x_1 - 50x_2/(1 + x_2)$	$400 - 50/(1 + x_2)$
Underflow product	50	$200x_1$	200
Overflow product	—	$1 - 50x_2/(1 + x_2)$	$1149 - 50/(1 + x_{2a})$
Thickener 2			
Underflow feed	50	$200x_1$	200
Overflow feed	—	$200x_2 - 50x_2/(1 + x_2)$	$400 - 50/(1 + x_2)$
Underflow product	50	$200x_2$	200
Overflow product	—	$200x_1 - 50x_2/(1 + x_2)$	$400 - 50/(1 + x_2)$

Again, assuming equilibrium is attained, then:

$$x_2 = [200x_1 - 50x_2/(1 + x_2)]/[400 - 50/(1 + x_2)]$$

and: $$x_1 = [1 - 50x_2/(1 + x_2)]/[1149 - 50/(1 + x_2)]$$

Solving simultaneously, then:

$$x_1 = 0.000870 \text{ Mg/Mg} \quad \text{and} \quad x_2 = 0.000435 \text{ Mg/Mg}$$

The solid product leaving the filter contains 50 Mg $CaCO_3$

and: $(50 \times 0.000435)/(1 + 0.000435) = 0.02175$ Mg NaOH in solution.

After drying, the solid product will contain:

$$(100 \times 50)/(50 + 0.02175) = \underline{\underline{99.96 \text{ per cent } CaCO_3}}$$

PROBLEM 10.7

Barium carbonate is to be made by reacting sodium carbonate and barium sulphide. The quantities fed to the reaction agitators per 24 hours are 20 Mg of barium sulphide dissolved in 60 Mg of water, together with the theoretically necessary amount of sodium carbonate.

Three thickeners in series are run on a countercurrent decantation system. Overflow from the second thickener goes to the agitators, and overflow from the first thickener is to contain 10 per cent sodium sulphide. Sludge from all thickeners contains two parts water to one part barium carbonate by mass. How much sodium sulphide will remain in the dried barium carbonate precipitate?

Solution

Basis: 1 day's operation

The reaction is:

$$BaS + Na_2CO_3 = BaCO_3 + Na_2S$$

Molecular masses: 169 106 197 78 kg/kmol.

Thus 20 Mg BaS will react to produce

$$(20 \times 197)/169 = 23.3 \text{ Mg } BaCO_3$$

and:

$$(20 \times 78)/169 = 9.23 \text{ Mg } Na_2S$$

The calculation may be made on the basis of this material entering the washing thickeners together with 60 Mg water. If x_1, x_2, and x_3 are the Na_2S/water ratio in the respective thickeners, then the mass balances are:

	$BaCO_3$	Na_2S	Water
Overall			
Underflow feed	23.3	9.23	60
Overflow feed	—	—	w (say)
Underflow product	23.3	$46.6x_3$	46.6
Overflow product	—	$9.23 - 46.6x_3$	$w + 13.4$

	$BaCO_3$	Na_2S	Water
Thickener 1			
Underflow feed	23.3	9.23	60
Overflow feed	—	$46.6(x_1 - x_3)$	w
Underflow product	23.3	$46.6x_1$	46.6
Overflow product	—	$9.23 - 46.6x_3$	$w + 13.4$
Thickener 2			
Underflow feed	23.3	$46.6x_1$	46.6
Overflow feed	—	$46.6(x_2 - x_3)$	w
Underflow product	23.3	$46.6x_2$	46.6
Overflow product	—	$46.6(x_1 - x_3)$	w
Thickener 3			
Underflow feed	23.3	$46.6x_2$	46.6
Overflow feed	—	—	w
Underflow product	23.3	$46.6x_3$	46.6
Overflow product	—	$46.6(x_2 - x_3)$	w

In the overflow product leaving the first thickener:

$$(9.23 - 46.6x_3)/(13.4 + w + 9.23 - 46.6x_3) = 0.10 \qquad \text{(i)}$$

Assuming equilibrium is attained in each thickener, then:

$$x_1 = (9.23 - 46.6x_3)/(13.4 + w), \qquad \text{(ii)}$$

$$x_2 = 46.6(x_1 - x_3)/w, \qquad \text{(iii)}$$

and:

$$x_3 = 46.6(x_2 - x_3)/w \qquad \text{(iv)}$$

Solving equations (i)–(iv) simultaneously:

$$x_1 = 0.112, \quad x_2 = 0.066, \quad x_3 = 0.030, \quad \text{and} \quad w = 57.1 \text{ Mg/day}$$

In the underflow product from the third thickener, the mass of Na_2S is

$$(46.6 \times 0.030) = 1.4 \text{ Mg associated with } 23.3 \text{ Mg } BaCO_3$$

When this stream is dried, the barium carbonate will contain:

$$(100 \times 1.4)/(1.4 + 23.3) = \underline{\underline{5.7 \text{ per cent sodium sulphide}}}$$

PROBLEM 10.8

In the production of caustic soda by the action of calcium hydroxide on sodium carbonate, 1 kg/s of sodium carbonate is treated with the theoretical quantity of lime. The sodium carbonate is made up as a 20 per cent solution. The material from the extractors is fed to a countercurrent washing system where it is treated with 2 kg/s of clean water. The washing thickeners are so arranged that the ratio of the volume of liquid discharged in the liquid offtake to that discharged with the solid is the same in all the thickeners and is equal to 4.0. How many thickeners must be arranged in series so that not more than 1 per cent of the sodium hydroxide discharged with the solid from the first thickener is wasted?

Solution

The reaction is:

$$Na_2CO_3 + Ca(OH)_2 = 2NaOH + CaCO_3$$

Molecular masses: 106 74 80 100 kg/kmol.

Thus 1 kg/s Na_2CO_3 forms $(1 \times 80)/106 = 0.755$ kg/s NaOH

and: $(1 \times 100)/106 = 0.943$ kg/s $CaCO_3$

In a 20 per cent solution, 1 kg/s Na_2CO_3 is associated with $(1 - 0.20)/0.2 = 4.0$ kg/s water.

If x kg/s NaOH leaves in the underflow product from the first thickener, then $0.01x$ kg/s NaOH should leave in the underflow product from the nth thickener. The amount of NaOH in the overflow from the first thickener is then given from an overall balance as $= (0.755 - 0.01x)$ kg/s:

Since the volume of the overflow product is $4x$, the volume of solution in underflow product, then:

$$(0.755 - 0.01x) = 4x \quad \text{and} \quad x = 0.188 \text{ kg/s}$$

and the NaOH leaving the nth thickener in the underflow $= (0.01 \times 0.188) = 0.00188$ kg/s.

Thus the fraction of solute fed to the washing system which remains associated with the washed solids, $f = (0.00188/0.755) = 0.0025$ kg/kg

In this case $R = 4.0$ and in equation 10.17:

$$n = \{\ln[1 + (4 - 1)/0.0025]\}/(\ln 4) - 1 = 4.11, \quad \text{say } \underline{\underline{5 \text{ washing thickeners}}}$$

PROBLEM 10.9

A plant produces 100 kg/s of titanium dioxide pigment which must be 99 per cent pure when dried. The pigment is produced by precipitation and the material, as prepared, is contaminated with 1 kg of salt solution containing 0.55 kg of salt/kg of pigment. The material is washed countercurrently with water in a number of thickeners arranged in series. How many thickeners will be required if water is added at the rate of 200 kg/s and the solid discharged from each thickeners removes 0.5 kg of solvent/kg of pigment?

What will be the required number of thickeners if the amount of solution removed in association with the pigment varies with the concentration of the solution in the thickener as follows:

kg solute/kg solution	0	0.1	0.2	0.3	0.4	0.5
kg solution/kg pigment	0.30	0.32	0.34	0.36	0.38	0.40

The concentrated wash liquor is mixed with the material fed to the first thickener.

Solution

Part I

The overall balance in kg/s, is:

	TiO_2	Salt	Water
Feed from reactor	100	55	45
Wash liquor added	—	—	200
Washed solid	100	0.1	50
Liquid product	—	54.9	195

The solvent in the underflow from the final washing thickener = 50 kg/s.

The solvent in the overflow will be the same as that supplied for washing = 200 kg/s.

This: $\dfrac{\text{Solvent discharged in overflow}}{\text{Solvent discharged in underflow}} = (200/50) = 4$ for the washing thickeners.

The liquid product from plant contains 54.9 kg of salt in 195 kg of solvent. This ratio will be the same in the underflow from the first thickener.

Thus the material fed to the washing thickeners consists of 100 kg TiO_2, 50 kg solvent and $(50 \times 54.9)/195 = 14$ kg salt.

The required number of thickeners for washing is given by equation 10.16 as:

$$(4 - 1)/(4^{n+1} - 1) = (0.1/14)$$

or: $$4^{n+1} = 421 \quad \text{giving:} \quad 4 < (n+1) < 5$$

Thus 4 washing thickeners or a total of <u>5 thickeners</u> are required.

Part 2

The same nomenclature will be used as in Volume 2, Chapter 10.

By inspection of the data, it is seen that $W_{h+1} = 0.30 + 0.2X_h$.

Thus: $$S_{h+1} = W_{h+1}X_h = 0.30X_h + 0.2X_h^2 = 5W_{h+1}^2 - 1.5W_{h+1}$$

Considering the passage of unit quantity of TiO_2 through the plant:

$$L_{n+1} = 0, \quad w_{n+1} = 2, \quad X_{n+1} = 0$$

since 200 kg/s pure solvent is used.

$$S_{n+1} = 0.001 \quad \text{and therefore} \quad W_{n+1} = 0.3007.$$

$$S_1 = 0.55 \quad \text{and} \quad W_1 = 1.00$$

Thus the concentration in the first thickener is given by equation 10.23:

$$X_1 = \frac{L_{n+1} + S_1 - S_{n+1}}{W_{n+1} + W_1 - W_{n+1}} = (0 + 0.55 - 0.001)/(2 + 1 - 0.3007) = 0.203$$

From equation 10.26:

$$X_{h+1} = \frac{L_{n+1} - S_{n+1} + S_{h+1}}{W_{n+1} - W_{n+1} + W_{h+1}} = \frac{(0 - 0.001 + S_{h+1})}{(2 - 0.3007 + W_{h+1})} = \frac{(-0.001 + S_{h+1})}{(1.7 + W_{h+1})}$$

Since $X_1 = 0.203$, then $W_2 = (0.30 + 0.2 \times 0.203) = 0.3406$

and: $S_2 = (0.3406 \times 0.203) = 0.0691$

Thus: $X_2 = (0.0691 - 0.001)/(1.7 + 0.3406) = 0.0334$

Since $X_2 = 0.0334$, then $W_3 = (0.30 + 0.2 \times 0.0334) = 0.30668$

and: $S_2 = (0.3067 \times 0.0334) = 0.01025$

Thus: $X_3 = (0.01025 - 0.001)/(1.7 + 0.3067) = 0.00447$

Since $X_3 = 0.00447$, then $W_4 = 0.30089$ and $S_4 = 0.0013$

Hence, by the same method: $X_4 = 0.000150$

Since $X_4 = 0.000150$, then $W_5 = 0.30003$ and $S_5 = 0.000045$.

Thus S_5 is less than S_{n+1} and therefore 4 thickeners are required.

PROBLEM 10.10

Prepared cottonseed meats containing 35 per cent of extractable oil are fed to a continuous countercurrent extractor of the intermittent drainage type using hexane as the solvent. The extractor consists of ten sections and the section efficiency is 50 per cent. The entrainment, assumed constant, is 1 kg solution/kg solids. What will be the oil concentration in the outflowing solvent if the extractable oil content in the meats is to be reduced by 0.5 per cent by mass?

Solution

Basis: 100 kg inert cottonseed material

Mass of oil in underflow feed $= (100 \times 0.35)/(1 - 0.35) = 53.8$ kg.

In the underflow product from the plant, mass of inerts = 100 kg and hence mass of oil $= (100 \times 0.005)/(1 - 0.005) = 0.503$ kg.

This is in 100 kg solution and hence the mass of hexane in the underflow product = $(100 - 0.503) = 99.497$ kg.

The overall balance in terms of mass is:

	Inerts	Oil	Hexane
Underflow feed	100	53.8	—
Overflow feed	—	—	h (say)
Underflow product	100	0.503	99.497
Overflow product	—	53.297	$(h - 99.497)$

Since there are ten stages, each 50 per cent efficient, the system may be considered, as a first approximation as consisting of five theoretical stages each of 100 per cent efficiency, in which equilibrium is attained in each stage. On this basis, the underflow from stage 1 contains 100 kg solution in which the oil/hexane ratio = 53.297/(h − 99.497) and hence the amount of oil in this stream is:

$$S_1 = 100[1 - (h - 99.497)/(h - 46.2)] \text{ kg}$$

$$S_{n+1} = 0.503 \text{ kg}$$

With constant underflow, the amount of solution in the overflow from each stage is say, h kg and the solution in the underflow = 100 kg.

Thus: $$R = (h/100) = 0.01h$$

and in equation 10.16:

$$0.503/[100 - 100(h - 99.497)/(h - 46.2)] = (0.01h - 1)/[(0.01h)^5 - 1]$$

or: $$(0.503h - 23.24) = (53.30h - 5330)/[(0.01h)^5 - 1]$$

Solving by trial and error: $h = 238$ kg
and in the overflow product:

mass of hexane = (238 − 99.497) = 138.5 kg, mass of oil = 53.3 kg

and concentration of oil = (100 × 53.3)/(53.3 + 138.5) = 27.8 per cent.

PROBLEM 10.11

Seeds containing 25 per cent by mass of oil are extracted in a countercurrent plant and 90 per cent of the oil is to be recovered in a solution containing 50 per cent of oil. It has been found that the amount of solution removed in the underflow in association with every kilogram of insoluble matter, k is given by:

$$k = 0.7 + 0.5y_s + 3y_s^2 \text{ kg/kg}$$

where y_s is the concentration of the overflow solution in terms of mass fraction of solute kg/kg. If the seeds are extracted with fresh solvent, how many ideal stages are required?

Solution

Basis: 100 kg underflow feed to the first stage

The first step is to obtain the underflow line, that is a plot of x_s against x_A. The calculations are made as follows:

y_s	k	Ratio (kg/kg inerts)			Mass fraction	
		oil (ky_s)	solvent $k(1 - y_s)$	underflow ($k + 1$)	oil (x_A)	solvent (x_s)
0	0.70	0	0.70	1.70	0	0.412
0.1	0.78	0.078	0.702	1.78	0.044	0.394

y_s	k	Ratio (kg/kg inerts)			Mass fraction	
		oil (ky_s)	solvent $k(1 - y_s)$	underflow $(k + 1)$	oil (x_A)	solvent (x_s)
0.2	0.92	0.184	0.736	1.92	0.096	0.383
0.3	1.12	0.336	0.784	2.12	0.159	0.370
0.4	1.38	0.552	0.828	2.38	0.232	0.348
0.5	1.70	0.850	0.850	2.70	0.315	0.315
0.6	2.08	1.248	0.832	3.08	0.405	0.270
0.7	2.52	1.764	0.756	3.52	0.501	0.215
0.8	3.02	2.416	0.604	4.02	0.601	0.150
0.9	3.58	3.222	0.358	4.58	0.704	0.078
1.0	4.20	4.20	0	5.20	0.808	0

A plot of x_A against x_s is shown in Figure 10a.

Considering the *underflow feed*, the seeds contain 25 per cent oil and 75 per cent inerts, and the point $x_{s1} = 0$, $x_{A1} = 0.25$ is drawn in as x_1.

In the *overflow feed*, pure solvent is used and hence:

$$y_{s\cdot n+1} = 1.0, \quad y_{A\cdot n+1} = 0$$

This point is marked as y_{n+1}.

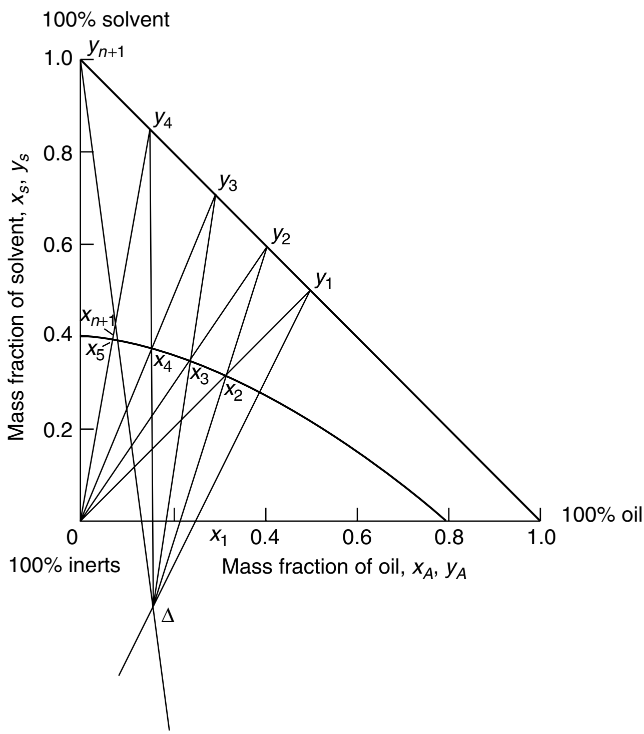

Figure 10a. Graphical construction for Problem 10.11

In the *overflow product*, the oil concentration is 50 per cent and $y_{s1} = 0.50$ and $y_{A1} = 0.50$. This point lies on the hypotenuse and is marked y_1.

90 per cent of the oil is recovered, leaving $25(1 - 0.90) = 2.5$ kg in the underflow product associated with 75 kg inerts; that is:

$$\text{ratio (oil/inerts)} = (2.5/75) = 0.033 = ky_s$$

Thus: $$0.033 = (0.7y_s + 0.5y_s^2 + 3y_s^3)$$

Solving by substitution gives:

$$y_s = 0.041 \quad \text{and hence} \quad k = (0.033/0.041) = 0.805$$

$$x_A = 0.0173 \quad \text{and} \quad x_s = 0.405$$

This point is drawn as x_{n+1} on the x_s against x_A curve.

The pole point Δ is obtained where $y_{n+1} \cdot x_{n+1}$ and $y_1 \cdot x_1$ extended meet and the construction described in Chapter 10 is then followed.

It is then found that x_{n+1} lies between x_4 and x_5 and hence 4 ideal stages are required.

PROBLEM 10.12

Halibut oil is extracted from granulated halibut livers in a countercurrent multi-batch arrangement using ether as the solvent. The solids charge contains 0.35 kg oil/kg of exhausted livers and it is desired to obtain a 90 per cent oil recovery. How many theoretical stages are required if 50 kg of ether are used/100 kg of untreated solids. The entrainment data are:

Concentration of overflow (kg oil/kg solution)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.67
Entrainment (kg solution/ kg extracted livers)	0.28	0.34	0.40	0.47	0.55	0.66	0.80	0.96

Solution

See Volume 2, Example 10.5.