

## *Humidification and Water Cooling*

### 13.1. INTRODUCTION

In the processing of materials it is often necessary either to increase the amount of vapour present in a gas stream, an operation known as *humidification*; or to reduce the vapour present, a process referred to as *dehumidification*. In humidification, the vapour content may be increased by passing the gas over a liquid which then evaporates into the gas stream. This transfer into the main stream takes place by diffusion, and at the interface simultaneous heat and mass transfer take place according to the relations considered in previous chapters. In the reverse operation, that is dehumidification, partial condensation must be effected and the condensed vapour removed.

The most widespread application of humidification and dehumidification involves the air-water system, and a discussion of this system forms the greater part of the present chapter. Although the drying of wet solids is an example of a humidification operation, the reduction of the moisture content of the solids is the main objective, and the humidification of the air stream is a secondary effect. Much of the present chapter is, however, of vital significance in any drying operation. Air conditioning and gas drying also involve humidification and dehumidification operations. For example, moisture must be removed from wet chlorine so that the gas can be handled in steel equipment which otherwise would be severely corroded. Similarly, the gases used in the manufacture of sulphuric acid must be dried or dehumidified before entering the converters, and this is achieved by passing the gas through a dehydrating agent such as sulphuric acid, in essence an absorption operation, or by an alternative dehumidification process discussed later.

In order that hot condenser water may be re-used in a plant, it is normally cooled by contact with an air stream. The equipment usually takes the form of a tower in which the hot water is run in at the top and allowed to flow downwards over a packing against a countercurrent flow of air which enters at the bottom of the cooling tower. The design of such towers forms an important part of the present chapter, though at the outset it is necessary to consider basic definitions of the various quantities involved in humidification, in particular *wet-bulb* and *adiabatic saturation temperatures*, and the way in which humidity data are presented on charts and graphs. While the present discussion is devoted to the very important air-water system, which is in some ways unique, the same principles may be applied to other liquids and gases, and this topic is covered in a final section.

## 13.2. HUMIDIFICATION TERMS

### 13.2.1. Definitions

The more important terms used in relation to humidification are defined as follows:

Humidity ( $\mathcal{H}$ )	mass of vapour associated with unit mass of dry gas
Humidity of saturated gas ( $\mathcal{H}_0$ )	humidity of the gas when it is saturated with vapour at a given temperature
Percentage humidity	$100(\mathcal{H}/\mathcal{H}_0)$
Humid heat ( $s$ )	heat required to raise unit mass of dry gas and its associated vapour through unit temperature difference at constant pressure, or: $s = C_a + \mathcal{H}C_w$ where $C_a$ and $C_w$ are the specific heat capacities of the gas and the vapour, respectively. (For the air-water system, the humid heat is approximately: $s = 1.00 + 1.9\mathcal{H}$ kJ/kg K.)
Humid volume	volume occupied by unit mass of dry gas and its associated vapour
Saturated volume	humid volume of saturated gas
Dew point	temperature at which the gas is saturated with vapour. As a gas is cooled, the dew point is the temperature at which condensation will first occur.
Percentage relative humidity	$\left( \frac{\text{partial pressure of vapour in gas}}{\text{partial pressure of vapour in saturated gas}} \right) \times 100$

The above nomenclature conforms with the recommendations of BS1339<sup>(1)</sup>, although there are some ambiguities in the standard.

The relationship between the partial pressure of the vapour and the humidity of a gas may be derived as follows. In unit volume of gas:

$$\text{mass of vapour} = \frac{P_w M_w}{RT}$$

$$\text{and mass of non-condensable gas} = \frac{(P - P_w)M_A}{RT}$$

The humidity is therefore given by:

$$\mathcal{H} = \frac{P_w}{P - P_w} \left( \frac{M_w}{M_A} \right) \quad (13.1)$$

and the humidity of the saturated gas is:

$$\mathcal{H}_0 = \frac{P_{w0}}{P - P_{w0}} \left( \frac{M_w}{M_A} \right) \quad (13.2)$$

where  $P_w$  is the partial pressure of vapour in the gas,  $P_{w0}$  the partial pressure of vapour in the saturated gas at the same temperature,  $M_A$  the mean molecular weight of the dry gas,  $M_w$  the molecular mass of the vapour,  $P$  the total pressure,  $R$  the gas constant (8314 J/kmol K in SI units), and  $T$  the absolute temperature.

For the air–water system,  $P_w$  is frequently small compared with  $P$  and hence, substituting for the molecular masses:

$$\mathcal{H} = \frac{18}{29} \left( \frac{P_w}{P} \right)$$

The relationship between the percentage humidity of a gas and the percentage relative humidity may be derived as follows:

The percentage humidity, by definition =  $100\mathcal{H}/\mathcal{H}_0$

Substituting from equations 13.1 and 13.2 and simplifying:

$$\begin{aligned} \text{Percentage humidity} &= \left( \frac{P - P_{w0}}{P - P_w} \right) \cdot \left( \frac{P_w}{P_{w0}} \right) \times 100 \\ &= \frac{(P - P_{w0})}{(P - P_w)} \times (\text{percentage relative humidity}) \quad (13.3) \end{aligned}$$

When  $(P - P_{w0})/(P - P_w) \approx 1$ , the percentage relative humidity and the percentage humidity are equal. This condition is approached when the partial pressure of the vapour is only a small proportion of the total pressure or when the gas is almost saturated, that is as  $P_w \rightarrow P_{w0}$ .

### Example 13.1

In a process in which it is used as a solvent, benzene is evaporated into dry nitrogen. At 297 K and 101.3 kN/m<sup>2</sup>, the resulting mixture has a percentage relative humidity of 60. It is required to recover 80 per cent of the benzene present by cooling to 283 K and compressing to a suitable pressure. What should this pressure be? The vapour pressure of benzene is 12.2 kN/m<sup>2</sup> at 297 K and 6.0 kN/m<sup>2</sup> at 283 K.

### Solution

From the definition of percentage relative humidity (RH):

$$P_w = P_{w0} \left( \frac{RH}{100} \right)$$

$$\text{At 297 K:} \quad P_w = (12.2 \times 1000) \times \left( \frac{60}{100} \right) = 7320 \text{ N/m}^2$$

In the benzene–nitrogen mixture:

$$\text{mass of benzene} = \frac{P_w M_w}{RT} = \frac{(7320 \times 78)}{(8314 \times 297)} = 0.231 \text{ kg}$$

$$\text{mass of nitrogen} = \frac{(P - P_w) M_A}{RT} = \frac{[(101.3 - 7320) \times 1000 \times 28]}{(8314 \times 297)} = 1.066 \text{ kg}$$

$$\text{Hence the humidity is:} \quad \mathcal{H} = \left( \frac{0.231}{1.066} \right) = 0.217 \text{ kg/kg}$$

In order to recover 80 per cent of the benzene, the humidity must be reduced to 20 per cent of the initial value. As the vapour will be in contact with liquid benzene, the nitrogen will be saturated with benzene vapour

and hence at 283 K:

$$\mathcal{H}_0 = \frac{(0.217 \times 20)}{100} = 0.0433 \text{ kg/kg}$$

Thus in equation 13.2:

$$0.0433 = \left( \frac{6000}{P - 6000} \right) \left( \frac{78}{28} \right)$$

from which:

$$\underline{\underline{P = 3.92 \times 10^5 \text{ N/m}^2 = 392 \text{ kN/m}^2}}$$

### Example 13.2

In a vessel at 101.3 kN/m<sup>2</sup> and 300 K, the percentage relative humidity of the water vapour in the air is 25. If the partial pressure of water vapour when air is saturated with vapour at 300 K is 3.6 kN/m<sup>2</sup>, calculate:

- the partial pressure of the water vapour in the vessel;
- the specific volumes of the air and water vapour;
- the humidity of the air and humid volume; and
- the percentage humidity.

### Solution

(a) From the definition of percentage relative humidity:

$$P_w = P_{ws} \frac{RH}{100} = 3600 \times \left( \frac{25}{100} \right) = 900 \text{ N/m}^2 = \underline{\underline{0.9 \text{ kN/m}^2}}$$

(b) In 1 m<sup>3</sup> of air:

$$\text{mass of water vapour} = \frac{(900 \times 18)}{(8314 \times 300)} = 0.0065 \text{ kg}$$

$$\text{mass of air} = \frac{[(101.3 - 0.9) \times 1000 \times 29]}{(8314 \times 300)} = 1.167 \text{ kg}$$

$$\text{Hence: specific volume of water vapour at } 0.9 \text{ kN/m}^2 = \left( \frac{1}{0.0065} \right) = \underline{\underline{154 \text{ m}^3/\text{kg}}}$$

$$\text{specific volume of air at } 100.4 \text{ kN/m}^2 = \left( \frac{1}{1.167} \right) = \underline{\underline{0.857 \text{ m}^3/\text{kg}}}$$

$$\text{(c) Humidity: } \mathcal{H} = \left( \frac{0.0065}{1.1673} \right) = \underline{\underline{0.0056 \text{ kg/kg}}}$$

(Using the approximate relationship:

$$\mathcal{H} = \frac{(18 \times 900)}{(29 \times 101.3 \times 1000)} = 0.0055 \text{ kg/kg.})$$

∴ Humid volume = volume of 1 kg air + associated vapour = specific volume of air at 100.4 kN/m<sup>2</sup>

$$= \underline{\underline{0.857 \text{ m}^3/\text{kg}}}$$

(d) From equation 13.3:

$$\begin{aligned} \text{percentage humidity} &= \frac{[(101.3 - 3.6) \times 1000]}{[(101.3 - 0.9) \times 1000]} \times 25 \\ &= \underline{\underline{24.3 \text{ per cent}}} \end{aligned}$$

### 13.2.2. Wet-bulb temperature

When a stream of unsaturated gas is passed over the surface of a liquid, the humidity of the gas is increased due to evaporation of the liquid. The temperature of the liquid falls below that of the gas and heat is transferred from the gas to the liquid. At equilibrium the rate of heat transfer from the gas just balances that required to vaporise the liquid and the liquid is said to be at the *wet-bulb temperature*. The rate at which this temperature is reached depends on the initial temperatures and the rate of flow of gas past the liquid surface. With a small area of contact between the gas and the liquid and a high gas flowrate, the temperature and the humidity of the gas stream remain virtually unchanged.

The rate of transfer of heat from the gas to the liquid can be written as:

$$Q = hA(\theta - \theta_w) \quad (13.4)$$

where  $Q$  is the heat flow,  $h$  the coefficient of heat transfer,  $A$  the area for transfer, and  $\theta$  and  $\theta_w$  are the temperatures of the gas and liquid phases.

The liquid evaporating into the gas is transferred by diffusion from the interface to the gas stream as a result of a concentration difference ( $c_0 - c$ ), where  $c_0$  is the concentration of the vapour at the surface (mass per unit volume) and  $c$  is the concentration in the gas stream. The rate of evaporation is then given by:

$$W = h_D A (c_0 - c) = h_D A \frac{M_w}{RT} (P_{w0} - P_w) \quad (13.5)$$

where  $h_D$  is the coefficient of mass transfer.

The partial pressures of the vapour,  $P_w$  and  $P_{w0}$ , may be expressed in terms of the corresponding humidities  $\mathcal{H}$  and  $\mathcal{H}_w$  by equations 13.1 and 13.2.

If  $P_w$  and  $P_{w0}$  are small compared with  $P$ ,  $(P - P_w)$  and  $(P - P_{w0})$  may be replaced by a mean partial pressure of the gas  $P_A$  and:

$$\begin{aligned} W &= h_D A \frac{(\mathcal{H}_w - \mathcal{H})M_w}{RT} \cdot \left( P_A \frac{M_A}{M_w} \right) \\ &= h_D A \rho_A (\mathcal{H}_w - \mathcal{H}) \end{aligned} \quad (13.6)$$

where  $\rho_A$  is the density of the gas at the partial pressure  $P_A$ .

The heat transfer required to maintain this rate of evaporation is:

$$Q = h_D A \rho_A (\mathcal{H}_w - \mathcal{H}) \lambda \quad (13.7)$$

where  $\lambda$  is the latent heat of vaporisation of the liquid.

Thus, equating equations 13.4 and 13.7:

$$(\mathcal{H} - \mathcal{H}_w) = - \frac{h}{h_D \rho_A \lambda} (\theta - \theta_w) \quad (13.8)$$

Both  $h$  and  $h_D$  are dependent on the equivalent gas film thickness, and thus any decrease in the thickness, as a result of increasing the gas velocity for example, increases both  $h$  and  $h_D$ . At normal temperatures,  $(h/h_D)$  is virtually independent of the gas velocity provided this is greater than about 5 m/s. Under these conditions, heat transfer by convection from the gas stream is large compared with that from the surroundings by radiation and conduction.

The wet-bulb temperature  $\theta_w$  depends only on the temperature and the humidity of the gas and values normally quoted are determined for comparatively high gas velocities, such that the condition of the gas does not change appreciably as a result of being brought into contact with the liquid and the ratio  $(h/h_D)$  has reached a constant value. For the air-water system, the ratio  $(h/h_D \rho_A)$  is about 1.0 kJ/kg K and varies from 1.5 to 2.0 kJ/kg K for organic liquids.

### Example 13.3

Moist air at 310 K has a wet-bulb temperature of 300 K. If the latent heat of vaporisation of water at 300 K is 2440 kJ/kg, estimate the humidity of the air and the percentage relative humidity. The total pressure is 105 kN/m<sup>2</sup> and the vapour pressure of water vapour at 300 K is 3.60 kN/m<sup>2</sup> and 6.33 kN/m<sup>2</sup> at 310 K.

### Solution

The humidity of air saturated at the wet-bulb temperature is given by:

$$\begin{aligned} \mathcal{H}_w &= \frac{P_{w0}}{P - P_{w0}} \frac{M_w}{M_A} && \text{(equation 13.2)} \\ &= \left( \frac{3.6}{105.0 - 3.6} \right) \left( \frac{18}{29} \right) = 0.0220 \text{ kg/kg} \end{aligned}$$

Therefore, taking  $(h/h_D \rho_A)$  as 1.0 kJ/kg K, in equation 13.8:

$$(0.0220 - \mathcal{H}) = \left( \frac{1.0}{2440} \right) (310 - 300)$$

or:

$$\mathcal{H} = \underline{\underline{0.018 \text{ kg/kg}}}$$

At 310 K,

$$P_{w0} = 6.33 \text{ kN/m}^2$$

In equation 13.2:

$$0.0780 = \frac{18P_w}{(105.0 - P_w)29}$$

$$P_w = 2.959 \text{ kN/m}^2$$

and the percentage relative humidity  $= \frac{(100 \times 2.959)}{6.33} = \underline{\underline{46.7 \text{ per cent}}}$

### 13.2.3. Adiabatic saturation temperature

In the system just considered, neither the humidity nor the temperature of the gas is appreciably changed. If the gas is passed over the liquid at such a rate that the time of contact is sufficient for equilibrium to be established, the gas will become saturated and both phases will be brought to the same temperature. In a thermally insulated system, the total sensible heat falls by an amount equal to the latent heat of the liquid evaporated. As a result of continued passage of the gas, the temperature of the liquid gradually approaches an equilibrium value which is known as the *adiabatic saturation temperature*.

These conditions are achieved in an infinitely tall thermally insulated humidification column through which gas of a given initial temperature and humidity flows

countercurrently to the liquid under conditions where the gas is completely saturated at the top of the column. If the liquid is continuously circulated round the column, and if any fresh liquid which is added is at the same temperature as the circulating liquid, the temperature of the liquid at the top and bottom of the column, and of the gas at the top, approach the adiabatic saturation temperature. Temperature and humidity differences are a maximum at the bottom and zero at the top, and therefore the rates of transfer of heat and mass decrease progressively from the bottom to the top of the tower. This is illustrated in Figure 13.1.

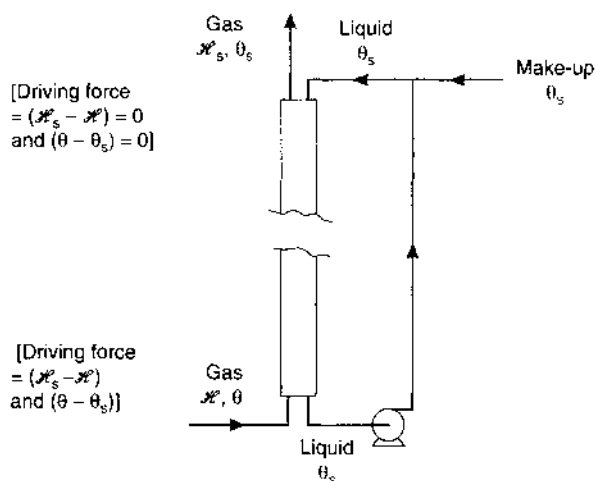


Figure 13.1. Adiabatic saturation temperature  $\theta_s$ .

Making a heat balance over the column, it is seen that the heat of vaporisation of the liquid must come from the sensible heat in the gas. The temperature of the gas falls from  $\theta$  to the adiabatic saturation temperature  $\theta_s$ , and its humidity increases from  $\mathcal{H}$  to  $\mathcal{H}_s$  (the saturation value at  $\theta_s$ ). Then working on the basis of unit mass of dry gas:

$$(\theta - \theta_s)s = (\mathcal{H}_s - \mathcal{H})\lambda$$

$$\text{or: } (\mathcal{H} - \mathcal{H}_s) = -\frac{s}{\lambda}(\theta - \theta_s) \quad (13.9)$$

where  $s$  is the humid heat of the gas and  $\lambda$  the latent heat of vaporisation at  $\theta_s$ .  $s$  is almost constant for small changes in  $\mathcal{H}$ .

Equation 13.9 indicates an approximately linear relationship between humidity and temperature for all mixtures of gas and vapour having the same adiabatic saturation temperature  $\theta_s$ . A curve of humidity versus temperature for gases with a given adiabatic saturation temperature is known as an *adiabatic cooling line*. For a range of adiabatic saturation temperatures, a family of curves, approximating to straight lines of slopes equal to  $-(s/\lambda)$ , is obtained. These lines are not exactly straight and parallel because of variations in  $\lambda$  and  $s$ .

Comparing equations 13.8 and 13.9, it is seen that the adiabatic saturation temperature is equal to the wet-bulb temperature when  $s = h/h_D\rho_A$ . This is the case for most water vapour systems and accurately so when  $\mathcal{H} = 0.047$ . The ratio  $(h/h_D\rho_A s) = b$  is sometimes known as the *psychrometric ratio* and, as indicated,  $b$  is approximately unity for the air-water system. For most systems involving air and an organic liquid,  $b = 1.3 - 2.5$  and the wet-bulb temperature is higher than the adiabatic saturation temperature. This was confirmed in 1932 by SHERWOOD and COMINGS<sup>(2)</sup> who worked with water, ethanol, n-propanol, n-butanol, benzene, toluene, carbon tetrachloride, and n-propyl acetate, and found that the wet-bulb temperature was always higher than the adiabatic saturation temperature except in the case of water.

In Chapter 12 it is shown that when the Schmidt and Prandtl numbers for a mixture of gas and vapour are approximately equal to unity, the *Lewis relation* applies, or:

$$h_D = \frac{h}{C_p\rho} \quad (\text{equation 12.105})$$

where  $C_p$  and  $\rho$  are the mean specific heat and density of the vapour phase.

Therefore: 
$$\frac{h}{h_D\rho_A} = \frac{C_p\rho}{\rho_A} \quad (13.10)$$

Where the humidity is relatively low,  $C_p \approx s$  and  $\rho \approx \rho_A$  and hence:

$$s \approx \frac{h}{h_D\rho_A} \quad (13.11)$$

For systems containing vapour other than that of water,  $s$  is only approximately equal to  $h/h_D\rho_A$  and the difference between the two quantities may be as high as 50 per cent.

If an unsaturated gas is brought into contact with a liquid which is at the adiabatic saturation temperature of the gas, a simultaneous transfer of heat and mass takes place. The temperature of the gas falls and its humidity increases (Figure 13.2). The temperature of the liquid at any instant tends to change and approach the wet-bulb temperature corresponding to the particular condition of the gas at that moment. For a liquid other than water, the adiabatic saturation temperature is less than the wet-bulb temperature and therefore in the initial stages, the temperature of the liquid rises. As the gas becomes humidified, however, its wet-bulb temperature falls and consequently the temperature to

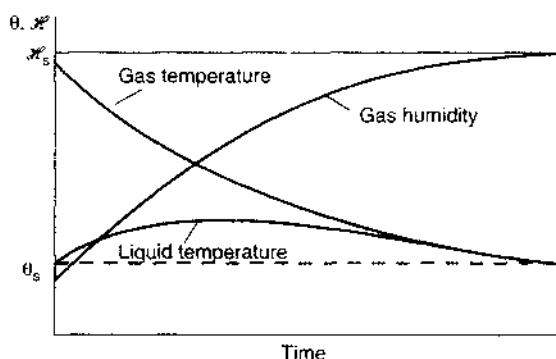


Figure 13.2. Saturation of gas with liquid other than water at the adiabatic saturation temperature



which the liquid is tending decreases as evaporation takes place. In due course, therefore, a point is reached where the liquid actually reaches the wet-bulb temperature of the gas in contact with it. It does not remain at this temperature, however, because the gas is not then completely saturated, and further humidification is accompanied by a continued lowering of the wet-bulb temperature. The temperature of the liquid therefore starts to fall and continues to fall until the gas is completely saturated. The liquid and gas are then both at the adiabatic saturation temperature.

The air-water system is unique, however, in that the Lewis relation holds quite accurately, so that the adiabatic saturation temperature is the same as the wet-bulb temperature. If, therefore, an unsaturated gas is brought into contact with water at the adiabatic saturation temperature of the gas, there is no tendency for the temperature of the water to change, and it remains in a condition of dynamic equilibrium through the whole of the humidification process (Figure 13.3). In this case, the adiabatic cooling line represents the conditions of constant wet-bulb temperatures as well as constant adiabatic saturation temperatures. The change in the condition of a gas as it is humidified with water vapour is therefore represented by the adiabatic cooling line and the intermediate conditions of the gas during the process are readily obtained. This is particularly useful because only partial humidification is normally obtained in practice.

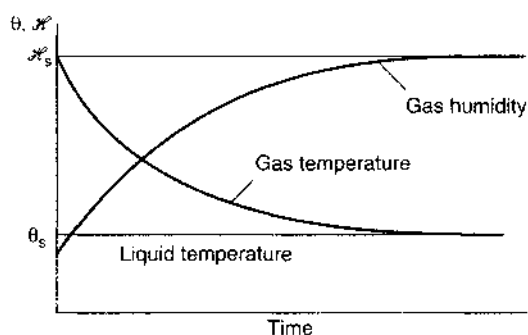


Figure 13.3. Saturation of air with water at adiabatic saturation temperature

### 13.3. HUMIDITY DATA FOR THE AIR-WATER SYSTEM

To facilitate calculations, various properties of the air-water system are plotted on a *psychrometric* or *humidity chart*. Such a chart is based on either the temperature or the enthalpy of the gas. The temperature-humidity chart is the more commonly used though the enthalpy-humidity chart is particularly useful for determining the effect of mixing two gases or of mixing a gas and a liquid. Each chart refers to a particular total pressure of the system. A humidity-temperature chart for the air-water system at atmospheric pressure, based on the original chart by GROSVENOR<sup>(3)</sup>, is given in Figure 13.4 and the corresponding humidity-enthalpy chart is given in Figure 13.5.

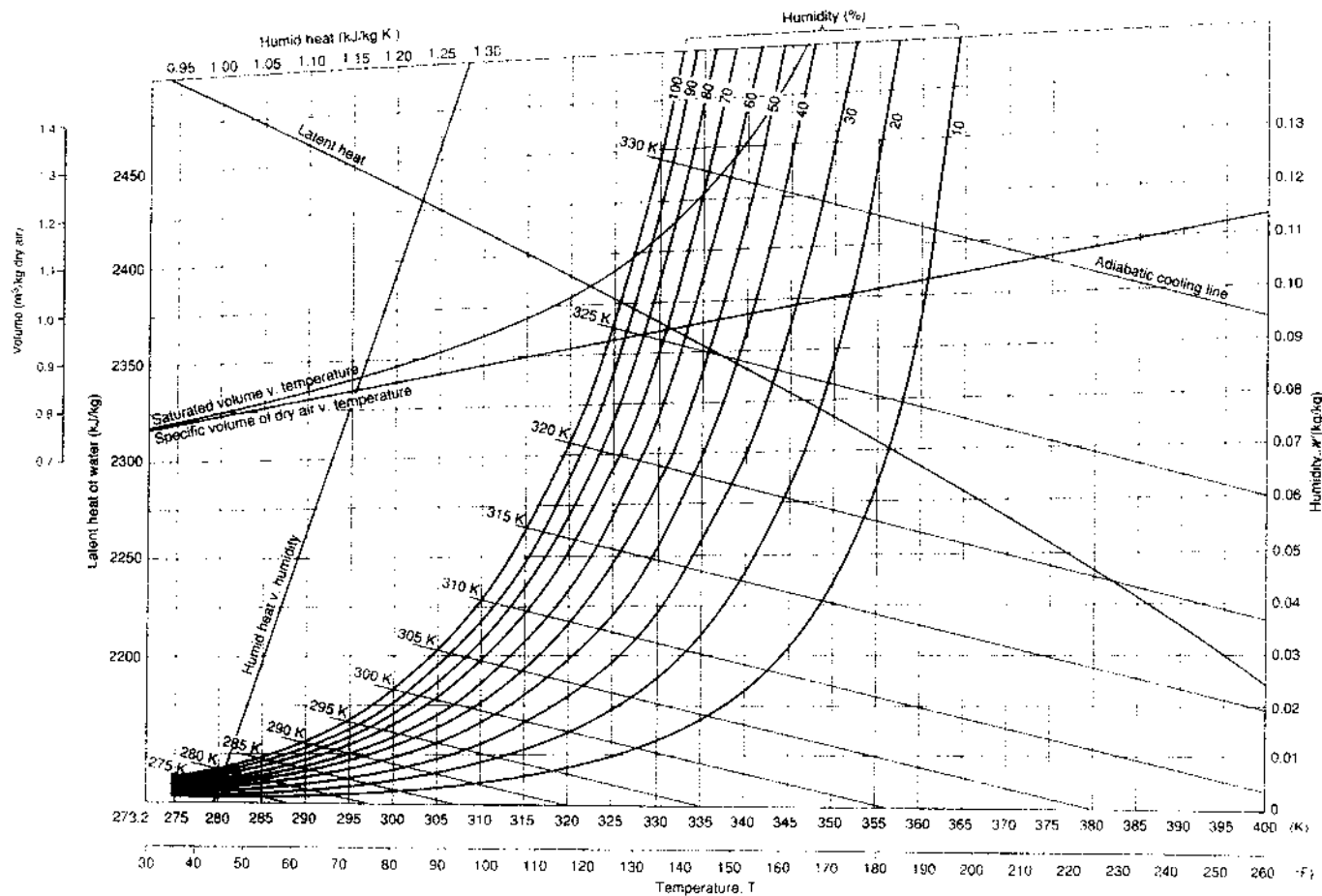


Figure 13.4. Humidity-temperature chart (See also the Appendix)

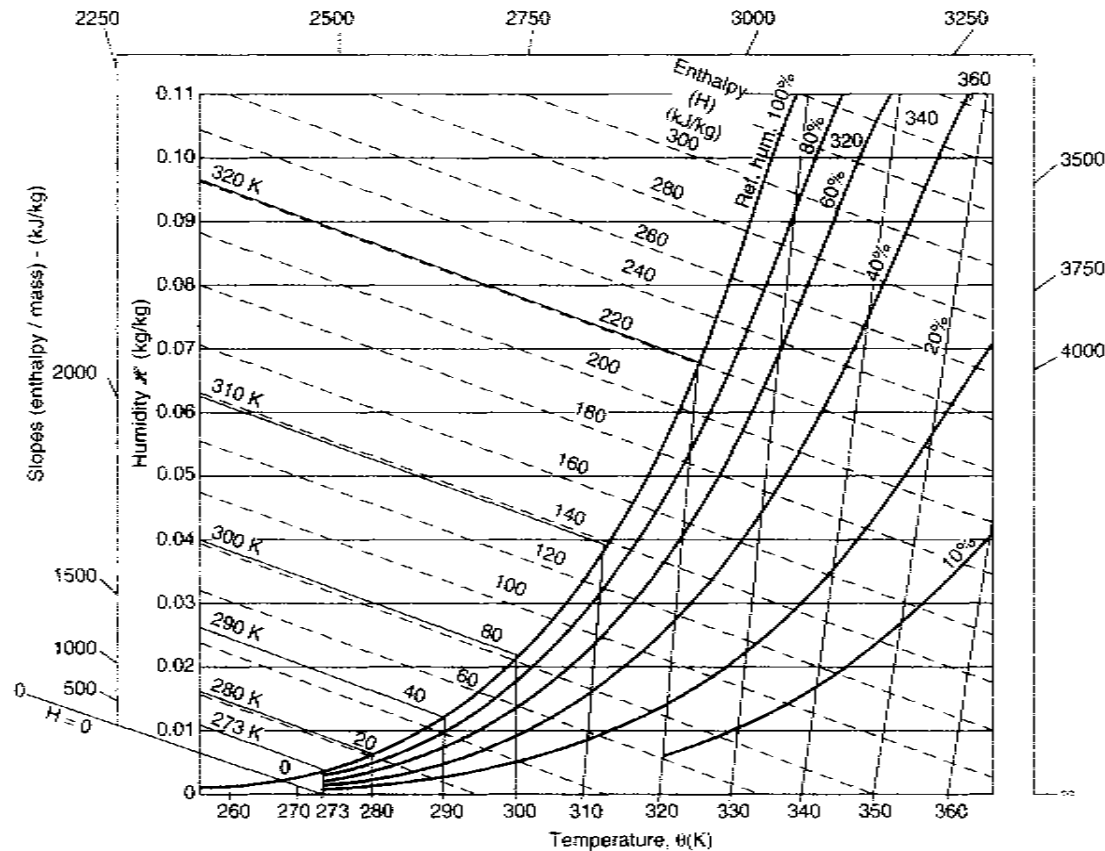


Figure 13.5. Humidity-enthalpy diagram for air-water vapour system at atmospheric pressure

### 13.3.1. Temperature–humidity chart

In Figure 13.4 it will be seen that the following quantities are plotted against temperature:

- (i) The *humidity*  $\mathcal{H}$  for various values of the percentage humidity.

For saturated gas: 
$$\mathcal{H}_0 = \frac{P_{w0}}{P - P_{w0}} \left( \frac{M_w}{M_A} \right) \quad (\text{equation 13.2})$$

From equation 13.1 for a gas with a humidity less than the saturation value:

$$\mathcal{H} = \frac{P_w}{P - P_w} \left( \frac{M_w}{M_A} \right) = \mathcal{H}_0 \frac{P_w}{P_{w0}} \frac{P - P_{w0}}{P - P_w} \quad (13.12)$$

- (ii) *The specific volume of dry gas.* This is a linear function of temperature.

(iii) *The saturated volume.* This increases more rapidly with temperature than the specific volume of dry gas because both the quantity and the specific volume of vapour increase with temperature. At a given temperature, the humid volume varies linearly with humidity and hence the humid volume of unsaturated gas can be found by interpolation.

- (iv) *The latent heat of vaporisation*

In addition, the *humid heat* is plotted as the abscissa in Figure 13.4 with the humidity as the ordinate.

Adiabatic cooling lines are included in the diagram and, as already discussed, these have a slope of  $-(s/\lambda)$  and they are slightly curved since  $s$  is a function of  $\mathcal{H}$ . On the chart they appear as straight lines, however, since the inclination of the axis has been correspondingly adjusted. Each adiabatic cooling line represents the composition of all gases whose adiabatic saturation temperature is given by its point of intersection with the 100 per cent humidity curve. For the air–water system, the adiabatic cooling lines represent conditions of constant wet-bulb temperature as well and, as previously mentioned, enable the change in composition of a gas to be followed as it is humidified by contact with water at the adiabatic saturation temperature of the gas.

#### Example 13.4

Air containing 0.005 kg water vapour per kg of dry air is heated to 325 K in a dryer and passed to the lower shelves. It leaves these shelves at 60 per cent humidity and is reheated to 325 K and passed over another set of shelves, again leaving at 60 per cent humidity. This is again repeated for the third and fourth sets of shelves, after which the air leaves the dryer. On the assumption that the material on each shelf has reached the wet-bulb temperature and that heat losses from the dryer may be neglected, determine:

- the temperature of the material on each tray;
- the amount of water removed in kg/s, if  $5 \text{ m}^3/\text{s}$  moist air leaves the dryer;
- the temperature to which the inlet air would have to be raised to carry out the drying in a single stage.

#### Solution

For each of the four sets of shelves, the condition of the air is changed to 60 per cent humidity along an adiabatic cooling line.

Initial condition of air:  $\theta = 325 \text{ K}, \mathcal{H} = 0.005 \text{ kg/kg}$

On humidifying to 60 per cent humidity:

$$t = 301 \text{ K}, \quad \mathcal{H} = 0.015 \text{ kg/kg and } \theta_w = 296 \text{ K}$$

At the end of the second pass:  $t = 308 \text{ K}, \quad \mathcal{H} = 0.022 \text{ kg/kg and } \theta_w = 301 \text{ K}$

At the end of the third pass:  $t = 312 \text{ K}, \quad \mathcal{H} = 0.027 \text{ kg/kg and } \theta_w = 305 \text{ K}$

At the end of the fourth pass:  $t = 315 \text{ K}, \quad \mathcal{H} = 0.032 \text{ kg/kg and } \theta_w = 307 \text{ K}$

Thus the temperatures of the material on each of the trays are:

$$\underline{\underline{296 \text{ K}, 301 \text{ K}, 305 \text{ K}, and } 307 \text{ K}}$$

$$\text{Total increase in humidity} = (0.032 - 0.005) = 0.027 \text{ kg/kg}$$

The air leaving the system is at 315 K and 60 per cent humidity.

From Figure 13.4, specific volume of dry air =  $0.893 \text{ m}^3/\text{kg}$

Specific volume of saturated air (*saturated volume*) =  $0.968 \text{ m}^3/\text{kg}$

Therefore, by interpolation, the humid volume of air of 60 per cent humidity =  $0.937 \text{ m}^3/\text{kg}$

$$\text{Mass of air passing through the dryer} = \left( \frac{5}{0.937} \right) = 5.34 \text{ kg/s}$$

$$\text{Mass of water evaporated} = (5.34 \times 0.027) = \underline{\underline{0.144 \text{ kg/s}}}$$

If the material is to be dried by air in a single pass, the air must be heated before entering the dryer such that its wet-bulb temperature is 307 K.

For air with a humidity of 0.005 kg/kg, this corresponds to a dry bulb temperature of 370 K.

The various steps in this calculation are shown in Figure 13.6.

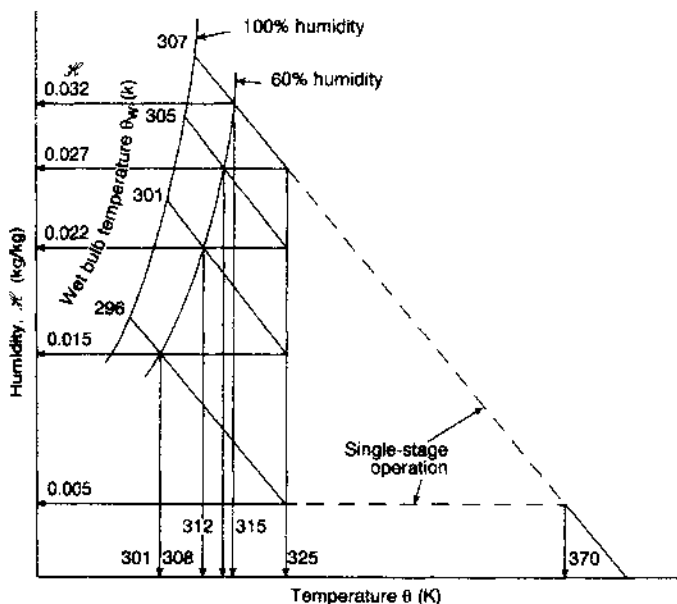


Figure 13.6. Humidification stages for Example 13.4 (schematic)

### 13.3.2. Enthalpy-humidity chart

In the calculation of enthalpies it is necessary to define some standard reference state at which the enthalpy is taken as zero. It is most convenient to take the melting point of the material constituting the vapour as the reference temperature, and the liquid state of the material as its standard state.

If  $H$  is the enthalpy of the humid gas per unit mass of dry gas,  $H_a$  the enthalpy of the dry gas per unit mass,  $H_w$  the enthalpy of the vapour per unit mass,  $C_a$  the specific heat of the gas at constant pressure,  $C_w$  the specific heat of the vapour at constant pressure,  $\theta$  the temperature of the humid gas,  $\theta_0$  the reference temperature,  $\lambda$  the latent heat of vaporisation of the liquid at  $\theta_0$  and  $\mathcal{H}$  the humidity of the gas.

$$\text{then for an unsaturated gas: } H = H_a + H_w \mathcal{H} \quad (13.13)$$

$$\text{where: } H_a = C_a(\theta - \theta_0) \quad (13.14)$$

$$\text{and: } H_w = C_w(\theta - \theta_0) + \lambda \quad (13.15)$$

$$\begin{aligned} \text{Thus, in equation 13.13: } H &= (C_a + \mathcal{H}C_w)(\theta - \theta_0) + \mathcal{H}\lambda \\ &= (\theta - \theta_0)(s + \mathcal{H}\lambda) \end{aligned} \quad (13.16)$$

If the gas contains more liquid or vapour than is required to saturate it at the temperature in question, either the gas will be supersaturated or the excess material will be present in the form of liquid or solid according to whether the temperature  $\theta$  is greater or less than the reference temperature  $\theta_0$ . The supersaturated condition is unstable and will not be considered further.

If the temperature  $\theta$  is greater than  $\theta_0$  and if the humidity  $\mathcal{H}$  is greater than the humidity  $\mathcal{H}_0$  of saturated gas, the enthalpy  $H$  per unit mass of dry gas is given by:

$$H = C_a(\theta - \theta_0) + \mathcal{H}_0[C_w(\theta - \theta_0) + \lambda] + C_L(\mathcal{H} - \mathcal{H}_0)(\theta - \theta_0) \quad (13.17)$$

where  $C_L$  is the specific heat of the liquid.

If the temperature  $\theta$  is less than  $\theta_0$ , the corresponding enthalpy  $H$  is given by:

$$H = C_w(\theta - \theta_0) + \mathcal{H}_0[C_w(\theta - \theta_0) + \lambda] + (\mathcal{H} - \mathcal{H}_0)[C_s(\theta - \theta_0) + \lambda_f] \quad (13.18)$$

where  $C_s$  is the specific heat of the solid and  $\lambda_f$  is the latent heat of freezing of the liquid, a negative quantity.

Equations 13.16 to 13.18 give the enthalpy in terms of the temperature and humidity of the humid gas for the three conditions:  $\theta = \theta_0$ ,  $\theta > \theta_0$ , and  $\theta < \theta_0$  respectively. Thus, given the percentage humidity and the temperature, the humidity may be obtained from Figure 13.4, the enthalpy calculated from equations 13.16, 13.17 or 13.18 and plotted against the humidity, usually with enthalpy as the abscissa. Such a plot is shown in Figure 13.7 for the air-water system, which includes the curves for 100 per cent humidity and for some lower value, say  $Z$  per cent.

Considering the nature of the isothermals for the three conditions dealt with previously, at constant temperature  $\theta$  the relation between enthalpy and humidity for an unsaturated gas is:

$$H = \text{constant} + [C_w(\theta - \theta_0) + \lambda]\mathcal{H} \quad (13.19)$$

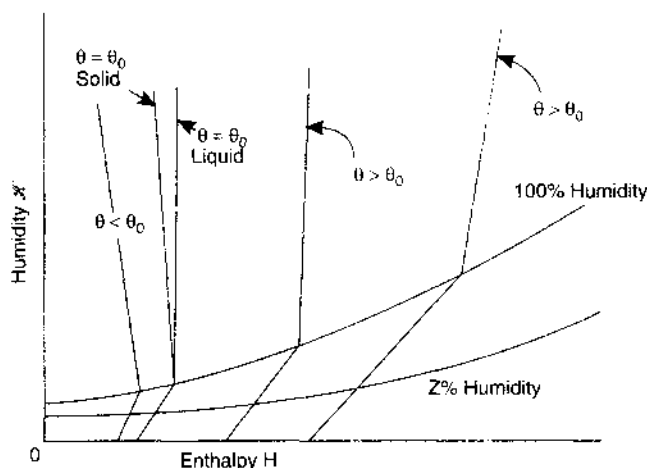


Figure 13.7. Humidity-enthalpy diagram for air-water system—rectangular axes

Thus, the isothermal is a straight line of slope  $[C_w(\theta - \theta_0) + \lambda]$  with respect to the humidity axis. At the reference temperature  $\theta_0$ , the slope is  $\lambda$ ; at higher temperatures, the slope is greater than  $\lambda$ , and at lower temperatures it is less than  $\lambda$ . Because the latent heat is normally large compared with the sensible heat, the slope of the isothermals remains positive down to very low temperatures. Since the humidity is plotted as the ordinate, the slope of the isothermal relative to the X-axis decreases with increase in temperature. When  $\theta > \theta_0$  and  $\mathcal{H} > \mathcal{H}_0$ , the saturation humidity, the vapour phase consists of a saturated gas with liquid droplets in suspension. The relation between enthalpy and humidity at constant temperature  $\theta$  is:

$$H = \text{constant} + C_L(\theta - \theta_0)\mathcal{H} \quad (13.20)$$

The isothermal is therefore a straight line of slope  $C_L(\theta - \theta_0)$ . At the reference temperature  $\theta_0$ , the slope is zero and the isothermal is parallel to the humidity axis. At higher temperatures, the slope has a small positive value. When  $\theta < \theta_0$  and  $\mathcal{H} > \mathcal{H}_0$ , solid particles are formed and the equation of the isothermal is:

$$H = \text{constant} + [C_s(\theta - \theta_0) + \lambda_f]\mathcal{H} \quad (13.21)$$

This represents a straight line of slope  $[C_s(\theta - \theta_0) + \lambda_f]$ . Both  $C_s(\theta - \theta_0)$  and  $\lambda_f$  are negative and therefore the slopes of all these isothermals are negative. When  $\theta = \theta_0$ , the slope is  $\lambda_f$ . In the supersaturated region therefore, there are two distinct isothermals at temperature  $\theta_0$ ; one corresponds to the condition where the excess vapour is present in the form of liquid droplets and the other to the condition where it is present as solid particles. The region between these isothermals represents conditions where a mixture of liquid and solid is present in the saturated gas at the temperature  $\theta_0$ .

The shape of the humidity-enthalpy line for saturated air is such that the proportion of the total area of the diagram representing saturated, as opposed to supersaturated, air is small when rectangular axes are used. In order to enable greater accuracy to be obtained in the use of the diagram, oblique axes are normally used, as in Figure 13.5. so

that the isothermal for unsaturated gas at the reference temperature  $\theta_0$  is parallel to the humidity axis.

It should be noted that the curves of humidity plotted against either temperature or enthalpy have a discontinuity at the point corresponding to the freezing point of the humidifying material. Above the temperature  $\theta_0$  the lines are determined by the vapour-liquid equilibrium and below it by the vapour-solid equilibrium.

Two cases may be considered to illustrate the use of enthalpy-humidity charts. These are the mixing of two streams of humid gas and the addition of liquid or vapour to a gas.

### Mixing of two streams of humid gas

Consider the mixing of two gases of humidities  $\mathcal{H}_1$  and  $\mathcal{H}_2$ , at temperatures  $\theta_1$  and  $\theta_2$ , and with enthalpies  $H_1$  and  $H_2$  to give a mixed gas of temperature  $\theta$ , enthalpy  $H$ , and humidity  $\mathcal{H}$ . If the masses of dry gas concerned are  $m_1$ ,  $m_2$ , and  $m$  respectively, then taking a balance on the dry gas, vapour, and enthalpy:

$$m_1 + m_2 = m \quad (13.22)$$

$$m_1\mathcal{H}_1 + m_2\mathcal{H}_2 = m\mathcal{H} \quad (13.23)$$

$$\text{and:} \quad m_1H_1 + m_2H_2 = mH \quad (13.24)$$

Elimination of  $m$  gives:

$$m_1(\mathcal{H} - \mathcal{H}_1) = m_2(\mathcal{H}_2 - \mathcal{H}) \quad (13.25)$$

$$\text{and:} \quad m_1(H - H_1) = m_2(H_2 - H)$$

Dividing these two equations:

$$\frac{(\mathcal{H} - \mathcal{H}_1)}{(H - H_1)} = \frac{(\mathcal{H}_2 - \mathcal{H})}{(H_2 - H)} \quad (13.26)$$

The condition of the resultant gas is therefore represented by a point on the straight line joining  $(\mathcal{H}_1, H_1)$  and  $(\mathcal{H}_2, H_2)$ . The humidity  $\mathcal{H}$  is given, from equation 13.25, by:

$$\frac{(\mathcal{H} - \mathcal{H}_1)}{(\mathcal{H}_2 - \mathcal{H})} = \frac{m_2}{m_1} \quad (13.27)$$

The gas formed by mixing two unsaturated gases may be either unsaturated, saturated, or supersaturated. The possibility of producing supersaturated gas arises because the 100 per cent humidity line on the humidity-enthalpy diagram is concave towards the humidity axis.

### Example 13.5

In an air-conditioning system, 1 kg/s air at 350 K and 10 per cent humidity is mixed with 5 kg/s air at 300 K and 30 per cent humidity. What is the enthalpy, humidity, and temperature of the resultant stream?

### Solution

From Figure 13.4:

$$\text{at } \theta_1 = 350 \text{ K and humidity} = 10 \text{ per cent: } \mathcal{H}_1 = 0.043 \text{ kg/kg}$$

$$\text{at } \theta_2 = 300 \text{ K and humidity} = 30 \text{ per cent: } \mathcal{H}_2 = 0.0065 \text{ kg/kg}$$



Thus, in equation 13.23:

$$(1 \times 0.043) + (5 \times 0.0065) = (1 + 5)\mathcal{H}$$

and:

$$\underline{\underline{\mathcal{H} = 0.0125 \text{ kg/kg}}}$$

From Figure 13.5:

$$\text{at } \theta_1 = 350 \text{ K and } \mathcal{H}_1 = 0.043 \text{ kg/kg: } H_1 = 192 \text{ kJ/kg}$$

$$\text{at } \theta_2 = 300 \text{ K and } \mathcal{H}_2 = 0.0065 \text{ kg/kg: } H_2 = 42 \text{ kJ/kg}$$

Thus, in equation 13.25:

$$1(H - 192) = 5(42 - H)$$

and:

$$\underline{\underline{H = 67 \text{ kJ/kg}}}$$

From Figure 13.5:

$$\text{at } H = 67 \text{ kJ/kg and } \mathcal{H} = 0.0125 \text{ kg/kg}$$

$$\underline{\underline{\theta = 309 \text{ K}}}$$

The data used in this example are shown in Figure 13.8.

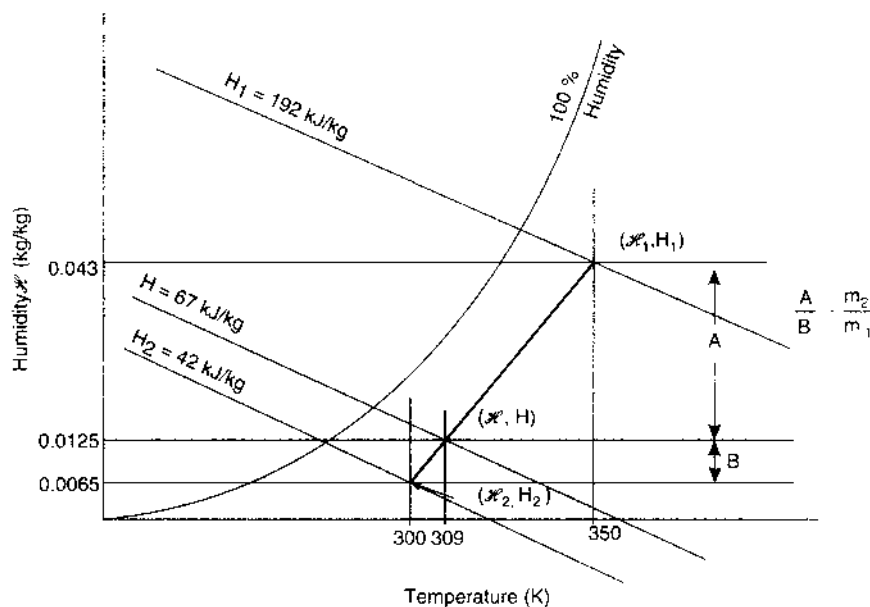


Figure 13.8. Data used in Example 13.5

### Addition of liquid or vapour to a gas

If a mass  $m_3$  of liquid or vapour of enthalpy  $H_3$  is added to a gas of humidity  $\mathcal{H}_1$  and enthalpy  $H_1$  and containing a mass  $m_1$  of dry gas, then:

$$m_1(\mathcal{H} - \mathcal{H}_1) = m_3 \quad (13.28)$$

$$m_1(H - H_1) = m_3 H_3 \quad (13.29)$$

Thus: 
$$\frac{(H - H_1)}{(\mathcal{H} - \mathcal{H}_1)} = H_3 \quad (13.30)$$

where  $\mathcal{H}$  and  $H$  are the humidity and enthalpy of the gas produced on mixing.

The composition and properties of the mixed stream are therefore represented by a point on the straight line of slope  $H_3$ , relative to the humidity axis, which passes through the point  $(H_1, \mathcal{H}_1)$ . In Figure 13.5 the edges of the plot are marked with points which, when joined to the origin, give a straight line of the slope indicated. Thus in using the chart, a line of slope  $H_3$  is drawn through the origin and a parallel line drawn through the point  $(H_1, \mathcal{H}_1)$ . The point representing the final gas stream is then given from equation 13.28:

$$(\mathcal{H} - \mathcal{H}_1) = \frac{m_3}{m_1}$$

It can be seen from Figure 13.5 that for the air-water system a straight line, of slope equal to the enthalpy of dry saturated steam (2675 kJ/kg), is almost parallel to the isothermals, so that the addition of live steam has only a small effect on the temperature of the gas. The addition of water spray, even if the water is considerably above the temperature of the gas, results in a lowering of the temperature after the water has evaporated. This arises because the latent heat of vaporisation of the liquid constitutes the major part of the enthalpy of the vapour. Thus, when steam is added, it gives up a small amount of sensible heat to the gas, whereas when hot liquid is added a small amount of sensible heat is given up and a very much larger amount of latent heat is absorbed from the gas.

### Example 13.6

0.15 kg/s steam at atmospheric pressure and superheated to 400 K is bled into an air stream at 320 K and 20 per cent relative humidity. What is the temperature, enthalpy, and relative humidity of the mixed stream if the air is flowing at 5 kg/s? How much steam would be required to provide an exit temperature of 330 K and what would be the humidity of this mixture?

### Solution

Steam at atmospheric pressure is saturated at 373 K at which the latent heat

$$= 2258 \text{ kJ/kg}$$

Taking the specific heat of superheated steam as 2.0 kJ/kg K;

$$\begin{aligned} \text{enthalpy of the steam: } H_3 &= 4.18(373 - 273) + 2258 + 2.0(400 - 373) \\ &= 2730 \text{ kJ/kg} \end{aligned}$$

From Figure 13.5:

$$\text{at } \theta_1 = 320 \text{ K and 20 per cent relative humidity: } \mathcal{H}_1 = 0.013 \text{ kg/kg and } H_1 = 83 \text{ kJ/kg}$$

The line joining the axis and slope  $H_3 = 2730 \text{ kJ/kg}$  at the edge of the chart is now drawn in and a parallel line is drawn through  $(H_1, \mathcal{H}_1)$ .

Thus: 
$$(\mathcal{H} - \mathcal{H}_1) = \frac{m_3}{m_1} = \left( \frac{0.15}{5} \right) = 0.03 \text{ kg/kg}$$

and: 
$$\mathcal{H} = (0.03 + 0.013) = 0.043 \text{ kg/kg}$$

At the intersection of  $\mathcal{H} = 0.043$  kg/kg and the line through  $(\mathcal{H}_1, H_1)$

$$\underline{H = 165 \text{ kJ/kg and } \theta = 324 \text{ K}}$$

When  $\theta = 330$  K the intersection of this isotherm and the line through  $(\mathcal{H}_1, H_1)$  gives an outlet stream in which  $\mathcal{H} = 0.094$  kg/kg (83 per cent relative humidity) and  $H = 300$  kJ/kg.

Thus, in equation 13.28:

$$\underline{m_3 = 5(0.094 - 0.013) = 0.41 \text{ kg/s}}$$

The data used in this example are shown in Figure 13.9.

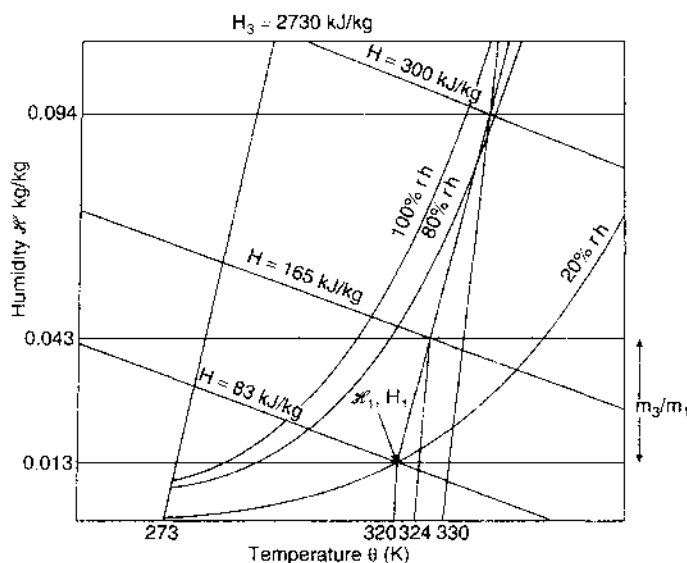


Figure 13.9. Data used in Example 13.6 (schematic)

### 13.4. DETERMINATION OF HUMIDITY

The most important methods for determining humidity are as follows:

(1) *Chemical methods.* A known volume of the gas is passed over a suitable absorbent, the increase in mass of which is measured. The efficiency of the process can be checked by arranging a number of vessels containing absorbent in series and ascertaining that the increase in mass in the last of these is negligible. The method is very accurate but is laborious. Satisfactory absorbents for water vapour are phosphorus pentoxide dispersed in pumice, and concentrated sulphuric acid.

(2) *Determination of the wet-bulb temperature.* Equation 13.8 gives the humidity of a gas in terms of its temperature, its wet-bulb temperature, and various physical properties of the gas and vapour. The wet-bulb temperature is normally determined as the temperature attained by the bulb of a thermometer which is covered with a piece of material which is maintained saturated with the liquid. The gas should be passed over the surface of the wet bulb at a high enough velocity ( $>5$  m/s) (a) for the condition of the gas stream not to be affected appreciably by the evaporation of liquid, (b) for the heat transfer by convection to be large compared with that by radiation and conduction from the surroundings, and

(c) for the ratio of the coefficients of heat and mass transfer to have reached a constant value. The gas should be passed long enough for equilibrium to be attained and, for accurate work, the liquid should be cooled nearly to the wet-bulb temperature before it is applied to the material.

The stream of gas over the liquid surface may be produced by a small fan or other similar means (Figure 13.10a). The crude forms of wet-bulb thermometer which make

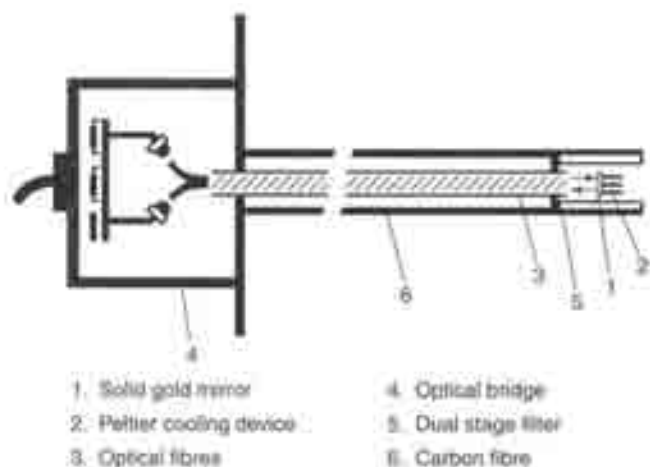


(a) Wet-bulb thermometer



(b) Dew-point meter

Figure 13.10. Hygrometers



(c) Dew-point meter with cyclic chilled-mirror system



(d) Hair hygrometer

Figure 13.10. (continued)

no provision for the rapid passage of gas cannot be used for accurate determinations of humidity.

(3) *Determination of the dew point.* The dew point is determined by cooling a highly polished surface in the gas and observing the highest temperature at which condensation takes place (Figure 13.10*b*). The humidity of the gas is equal to the humidity of saturated gas at the dew-point. The instrument illustrated in Figure 13.10*c* incorporates a polished gold mirror which is cooled using a thermo-electric module which utilises the *Peltier effect*.

(4) *Measurement of the change in length of a hair or fibre.* The length of a hair or fibre is influenced by the humidity of the surrounding atmosphere. Many forms of apparatus for automatic recording of humidity depend on this property. The method has the disadvantage that the apparatus needs frequent calibration because the zero tends to shift. This difficulty is most serious when the instrument is used over a wide range of humidities. A typical hair hygrometer is shown in Figure 13.10*d*.

(5) *Measurement of conductivity of a fibre.* If a fibre is impregnated with an electrolyte, such as lithium chloride, its electrical resistance will be governed by its moisture content, which in turn depends on the humidity of the atmosphere in which it is situated. In a lithium chloride cell, a skein of very fine fibres is wound on a plastic frame carrying the electrodes and the current flowing at a constant applied voltage gives a direct measure of the relative humidity.

(6) *Measurement of heat of absorption on to a surface.*

(7) *Electrolytic hygrometry* in which the quantity of electricity required to electrolyse water absorbed from the atmosphere on to a thin film of desiccant is measured.

(8) *Piezo-electric hygrometry* employing a quartz crystal with a hygroscopic coating in which moisture is alternately absorbed from a wet-gas and desorbed in a dry-gas stream; the dynamics is a function of the gas humidity.

(9) *Capacitance meters* in which the electrical capacitance is a function of the degree of deposition of moisture from the atmosphere.

(10) *Observation of colour changes* in active ingredients, such as cobaltous chloride.

Further details of instruments for the measurement of humidity are given in Volume 3. Reference should also be made to standard works on psychrometry<sup>(4,5,6)</sup>.

## 13.5. HUMIDIFICATION AND DEHUMIDIFICATION

### 13.5.1. Methods of increasing humidity

The following methods may be used for increasing the humidity of a gas:

(1) Live steam may be added directly in the required quantity. It has been shown that this produces only a slight increase in the temperature, but the method is not generally favoured because any impurities that are present in the steam may be added at the same time.

(2) Water may be sprayed into the gas at such a rate that, on complete vaporisation, it gives the required humidity. In this case, the temperature of the gas will fall as the latent heat of vaporisation must be supplied from the sensible heat of the gas and liquid.

(3) The gas may be mixed with a stream of gas of higher humidity. This method is frequently used in laboratory work when the humidity of a gas supplied to an apparatus is controlled by varying the proportions in which two gas streams are mixed.

(4) The gas may be brought into contact with water in such a way that only part of the liquid is evaporated. This is perhaps the most common method and will now be considered in more detail.

In order to obtain a high rate of humidification, the area of contact between the air and the water is made as large as possible by supplying the water in the form of a fine spray; alternatively, the interfacial area is increased by using a packed column. Evaporation occurs if the humidity at the surface is greater than that in the bulk of the air; that is, if the temperature of the water is above the dew point of the air.

When humidification is carried out in a packed column, the water which is not evaporated can be recirculated so as to reduce the requirements of fresh water. As a result of continued recirculation, the temperature of the water will approach the adiabatic saturation temperature of the air, and the air leaving the column will be cooled—in some cases to within 1 deg K of the temperature of the water. If the temperature of the air is to be maintained constant, or raised, the water must be heated.

Two methods of changing the humidity and temperature of a gas from  $A(\theta_1, \mathcal{H}_1)$  to  $B(\theta_2, \mathcal{H}_2)$  may be traced on the humidity chart as shown in Figure 13.11. The first method consists of saturating the air by water artificially maintained at the dew point of air of humidity  $\mathcal{H}_2$  (line AC) and then heating at constant humidity to  $\theta_2$  (line CB). In the second method, the air is heated (line AD) so that its adiabatic saturation temperature corresponds with the dew point of air of humidity  $\mathcal{H}_2$ . It is then saturated by water at the adiabatic saturation temperature (line DC) and heated at constant humidity to  $\theta_2$  (line CB). In this second method, an additional operation—the preliminary heating—is carried out on the air, but the water temperature automatically adjusts itself to the required value.

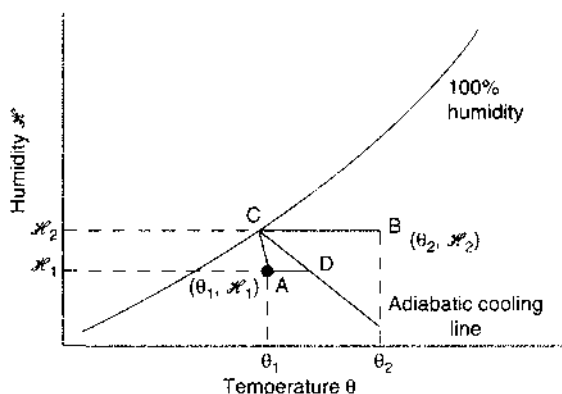


Figure 13.11. Two methods of changing conditions of gas from  $(\theta_1, \mathcal{H}_1)$  to  $(\theta_2, \mathcal{H}_2)$

Since complete humidification is not always attained, an allowance must be made when designing air humidification cycles. For example, if only 95 per cent saturation is attained the adiabatic cooling line should be followed only to the point corresponding to that degree of saturation, and therefore the gas must be heated to a slightly higher temperature before adiabatic cooling is commenced.

**Example 13.7**

Air at 300 K and 20 per cent humidity is to be heated in two stages with intermediate saturation with water to 90 per cent humidity so that the final stream is at 320 K and 20 per cent humidity. What is the humidity of the exit stream and the conditions at the end of each stage?

**Solution**

At  $\theta_1 = 300$  K and 20 per cent humidity:  $\mathcal{H}_1 = 0.0045$  kg/kg, from Figure 13.4, and

at  $\theta_2 = 320$  K and 20 per cent humidity:  $\mathcal{H}_2 = 0.0140$  kg/kg

When  $\mathcal{H}_2 = 0.0140$  kg/kg, air is saturated at 292 K and has a humidity of 90 per cent at 293 K.

The adiabatic cooling line corresponding to 293 K intersects with  $\mathcal{H} = 0.0045$  kg/kg at a temperature,  $\theta = 318$  K.

Thus the stages are:

- (i) Heat the air at  $\mathcal{H} = 0.0045$  from 300 to 318 K.
- (ii) Saturate with water at an adiabatic saturation temperature of 293 K until 90 per cent humidity is attained. At the end of this stage:

$$\mathcal{H} = 0.0140 \text{ kg/kg} \quad \text{and} \quad \theta = 294.5 \text{ K}$$

- (iii) Heat the saturated air at  $\mathcal{H} = 0.0140$  kg/kg from 294.5 to 320 K

**13.5.2. Dehumidification**

Dehumidification of air can be effected by bringing it into contact with a cold surface, either liquid or solid. If the temperature of the surface is lower than the dew point of the gas, condensation takes place and the temperature of the gas falls. The temperature of the surface tends to rise because of the transfer of latent and sensible heat from the air. It would be expected that the air would cool at constant humidity until the dew point was reached, and that subsequent cooling would be accompanied by condensation. It is found, in practice, that this occurs only when the air is well mixed. Normally the temperature and humidity are reduced simultaneously throughout the whole of the process. The air in contact with the surface is cooled below its dew point, and condensation of vapour therefore occurs before the more distant air has time to cool. Where the gas stream is cooled by cold water, countercurrent flow should be employed because the temperature of the water and air are changing in opposite directions.

The humidity can be reduced by compressing air, allowing it to cool again to its original temperature, and draining off the water which has condensed. During compression, the partial pressure of the vapour is increased and condensation takes place as soon as it reaches the saturation value. Thus, if air is compressed to a high pressure, it becomes saturated with vapour, but the partial pressure is a small proportion of the total pressure. Compressed air from a cylinder therefore has a low humidity. Gas is frequently compressed before it is circulated so as to prevent condensation in the mains.

Many large air-conditioning plants incorporate automatic control of the humidity and temperature of the issuing air. Temperature control is effected with the aid of a thermocouple or resistance thermometer, and humidity control by means of a thermocouple recording the difference between the wet- and dry-bulb temperatures.



## 13.6. WATER COOLING

### 13.6.1. Cooling towers

Cooling of water can be carried out on a small scale either by allowing it to stand in an open pond or by the spray pond technique in which it is dispersed in spray form and then collected in a large, open pond. Cooling takes place both by the transference of sensible heat and by evaporative cooling as a result of which sensible heat in the water provides the latent heat of vaporisation.

On the large scale, air and water are brought into countercurrent contact in a cooling tower which may employ either natural draught or mechanical draught. The water flows down over a series of wooden slats which give a large interfacial area and promote turbulence in the liquid. The air is humidified and heated as it rises, while the water is cooled mainly by evaporation.

The natural draught cooling tower depends on the chimney effect produced by the presence in the tower of air and vapour of higher temperature and therefore of lower density than the surrounding atmosphere. Thus atmospheric conditions and the temperature and quantity of the water will exert a very important effect on the operation of the tower. Not only will these factors influence the quantity of air drawn through the tower, but they will also affect the velocities and flow patterns and hence the transfer coefficients between gas and liquid. One of the prime considerations in design therefore is to construct a tower in such a way that the resistance to air flow is low. Hence the packings and distributors must be arranged in open formation. The draught of a cooling tower at full load is usually only about  $50 \text{ N/m}^2$ ,<sup>(7)</sup> and the air velocity in the region of  $1.2\text{--}1.5 \text{ m/s}$ , so that under the atmospheric conditions prevailing in the UK the air usually leaves the tower in a saturated condition. The density of the air stream at outlet is therefore determined by its temperature. Calculation of conditions within the tower is carried out in the manner described in the following pages. It is, however, necessary to work with a number of assumed air flowrates and to select the one which fits both the transfer conditions and the relationship between air rate and pressure difference in the tower.

The *natural draught cooling tower* consists of an empty shell, constructed either of timber or ferroconcrete, where the upper portion is empty and merely serves to increase the draught. The lower portion, amounting to about 10–12 per cent of the total height, is usually fitted with grids on to which the water is fed by means of distributors or sprays as shown in Figure 13.12. The shells of cooling towers are now generally constructed in ferroconcrete in a shape corresponding approximately to a hyperboloid of revolution. The shape is chosen mainly for constructional reasons, but it does take account of the fact that the entering air will have a radial velocity component; the increase in cross-section towards the top causes a reduction in the outlet velocity and there is a small recovery of kinetic energy into pressure energy.

The *mechanical draught cooling tower* may employ forced draught with the fan at the bottom, or induced draught with the fan driving the moist air out at the top. The air velocity can be increased appreciably above that in the natural draught tower, and a greater depth of packing can be used. The tower will extend only to the top of the packing unless atmospheric conditions are such that a chimney must be provided in order to prevent recirculation of the moist air. The danger of recirculation is considerably

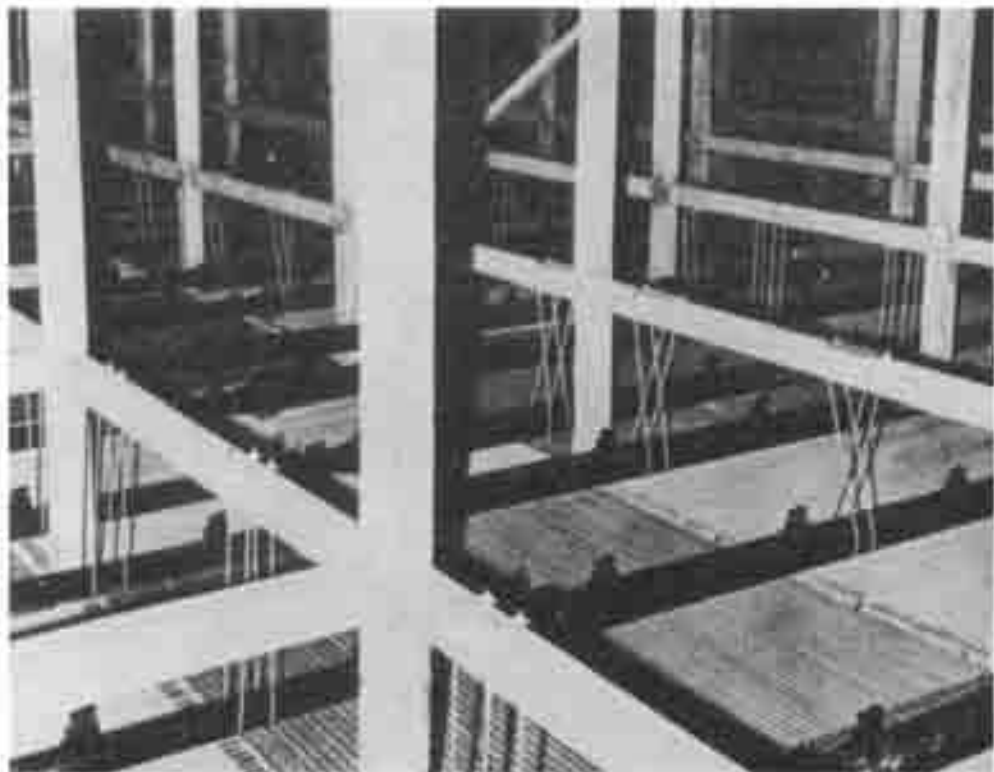
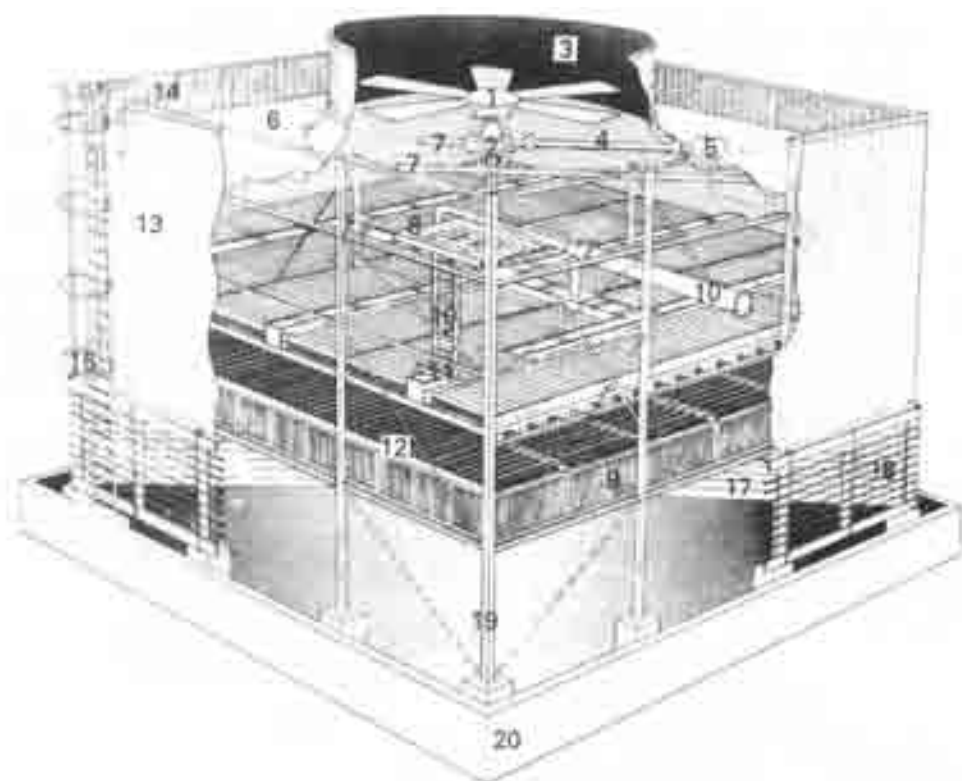


Figure 13.12. Water-cooling tower. View of spray distribution system

less with the induced-draught type because the air is expelled with a higher velocity. Mechanical draught towers are generally confined to small installations and to conditions where the water must be cooled to as low a temperature as possible. In some cases it is possible to cool the water to within 1 deg K of the wet-bulb temperature of the air. Although the initial cost of the tower is less, maintenance and operating costs are of course higher than in natural draught towers which are now used for all large installations. A typical steel-framed mechanical draught cooling tower is shown in Figure 13.13.

The operation of the conventional water cooling tower is often characterised by the discharge of a plume consisting of a suspension of minute droplets of water in air. This is formed when the hot humid air issuing from the top of the tower mixes with the ambient atmosphere, and precipitation takes place as described earlier (Section 13.3.2). In the *hybrid* (or wet/dry) cooling tower<sup>(8)</sup>, mist formation is avoided by cooling *part* of the water in a finned-tube exchanger bundle which thus generates a supply of warm dry air which is then blended with the air issuing from the evaporative section. By adjusting the proportion of the water fed to the heat exchanger, the plume can be completely eliminated.

In the cooling tower the temperature of the liquid falls and the temperature and humidity of the air rise, and its action is thus similar to that of an air humidifier. The limiting temperature to which the water can be cooled is the wet-bulb temperature corresponding to the condition of the air at inlet. The enthalpy of the air stream does not remain



- 1 Fan assembly
- 2 Gearbox
- 3 Fan stack
- 4 Drive shaft assembly
- 5 Motor
- 6 Fan deck
- 7 Mechanical equipment supports
- 8 Drift eliminators (PVC or Timber - Timber shown)
- 9 Cooling tower packing (plastic plate or wooden lath)
- 10 Inlet water distribution pipe
- 11 Open type distribution system
- 12 Timber laths for even water distribution
- 13 Cladding
- 14 Cladding extended to form handrail
- 15 Access ladder
- 16 Internal access ladder to distribution system and drift eliminators
- 17 Diagonal wind baffles
- 18 Air inlet louvres
- 19 Steel structures with horizontal and diagonal ties
- 20 Cold water sump

*Some structural members have been omitted for clarity*

Figure 13.13. Visco 2000 series steel framed, mechanical draught, water cooling tower

constant since the temperature of the liquid changes rapidly in the upper portion of the tower. Towards the bottom, however, the temperature of the liquid changes less rapidly because the temperature differences are smaller. At the top of the tower, the temperature falls from the bulk of the liquid to the interface and then again from the interface to the bulk of the gas. Thus the liquid is cooled by transfer of sensible heat and by evaporation at the surface. At the bottom of a tall tower, however, the temperature gradient in the liquid is in the same direction, though smaller, but the temperature gradient in the gas is in the opposite direction. Transfer of sensible heat to the interface therefore takes place from the bulk of the liquid and from the bulk of the gas, and all the cooling is caused by the evaporation at the interface. In most cases, about 80 per cent of the heat loss from the water is accounted for by evaporative cooling.

### 13.6.2. Design of natural-draught towers

The air flow through a natural-draught or hyperbolic-type tower (Figure 13.14) is due largely to the difference in density between the warm air in the tower and the external ambient air; thus a draught is created in the stack by a chimney effect which eliminates the need for mechanical fans. It has been noted by MCKELVEY and BROOKE<sup>(9)</sup> that natural-draught towers commonly operate at a pressure difference of some  $50 \text{ N/m}^2$  under full load, and above the packing the mean air velocity is typically  $1\text{--}2 \text{ m/s}$ . The performance of a natural-draught tower differs from that of a mechanical-draught installation in that the cooling achieved depends upon the relative humidity as well as the wet-bulb temperature. It is important therefore, at the design stage, to determine correctly, and to specify, the density of the inlet and exit air streams in addition to the usual tower design conditions of water temperature range, how closely the water temperature should approach the wet bulb temperature of the air, and the quantity of water to be handled. Because the performance



Figure 13.14. Natural draught water-cooling towers

depends to a large extent on atmospheric humidity, the outlet water temperature is difficult to control with natural-draught towers.

In the design of natural-draught towers, a ratio of height to base diameter of 3:2 is normally used and a design method has been proposed by CHILTON<sup>(10)</sup>. Chilton has shown that the duty coefficient  $D_t$  of a tower is approximately constant over the normal range of operation and is related to tower size by an efficiency factor or performance coefficient  $C_t$  given by:

$$D_t = \frac{19.50 A_b z_t^{0.5}}{C_t^{1.5}} \quad (13.31)$$

where for water loadings in excess of  $1 \text{ kg/m}^2\text{s}$ ,  $C_t$  is usually about 5.2 though lower values are obtained with new packings which are being developed.

The duty coefficient is given by the following equation (in which SI units must be used as it is not dimensionally consistent):

$$\frac{W_L}{D_t} = 0.00369 \frac{\Delta H'}{\Delta T} (\Delta T' + 0.0752 \Delta H')^{0.5} \quad (13.32)$$

where  $W_L$  (kg/s) is the water load in the tower,  $\Delta H'$  (kJ/kg) the change in enthalpy of the air passing through the tower,  $\Delta T$  (deg K) the change in water temperature in passing through the tower and  $\Delta T'$  (deg K), the difference between the temperature of the air leaving the packing and the dry-bulb temperature of the inlet air. The air leaving the packing inside the tower is assumed to be saturated at the mean of the inlet and outlet water temperatures. Any divergence between theory and practice of a few degrees in this respect does not significantly affect the results as the draught component depends on the ratio of the change of density to change in enthalpy and not on change in temperature alone.<sup>(11)</sup> The use of equations 13.31 and 13.32 is illustrated in the following example.

### Example 13.8

What are the diameter and height of a hyperbolic natural-draught cooling tower handling  $4810 \text{ kg/s}$  of water with the following temperature conditions:

water entering the tower =  $301 \text{ K}$

water leaving the tower =  $294 \text{ K}$ .

air: dry bulb =  $287 \text{ K}$

wet bulb =  $284 \text{ K}$

### Solution

Temperature range for the water,  $\Delta T = (301 - 294) = 7 \text{ deg K}$ .

At a mean water temperature of  $0.5(301 + 294) = 297.5 \text{ K}$ , the enthalpy =  $92.6 \text{ kJ/kg}$

At a dry bulb temperature of  $287 \text{ K}$ , the enthalpy =  $49.5 \text{ kJ/kg}$

$\therefore \Delta T' = (297.5 - 287) = 10.5 \text{ deg K}$

and:  $\Delta H' = (92.6 - 49.5) = 43.1 \text{ kJ/kg}$

In equation 13.32:

$$\frac{4810}{D_t} = 0.00369 \left( \frac{43.1}{7} \right) [10.5 + (0.0752 \times 43.1)]^{0.5}$$

and:  $D_t = 57.110$

Taking  $C_1$  as 5.0 and assuming as a first estimate a tower height of 100 m, then in equation 13.31,

$$57.110 = 19.504_b \frac{100^{0.5}}{5.0^{1.5}}$$

and

$$A_b = 3274 \text{ m}^2$$

$$\begin{aligned} \text{Thus the internal diameter of the column at sill level} &= \left( \frac{3274 \times 4}{\pi} \right)^{0.5} \\ &= \underline{\underline{64.6 \text{ m}}} \end{aligned}$$

Since this gives a height: diameter ratio of  $(100 : 64.6) \approx 3 : 2$ , the design is acceptable.

### 13.6.3. Height of packing for both natural and mechanical draught towers

The height of a water-cooling tower can be determined<sup>(12)</sup> by setting up a material balance on the water, an enthalpy balance, and rate equations for the transfer of heat in the liquid and gas and for mass transfer in the gas phase. There is no concentration gradient in the liquid and therefore there is no resistance to mass transfer in the liquid phase.

Considering the countercurrent flow of water and air in a tower of height  $z$  (Figure 13.15), the mass rate of flow of air per unit cross-section  $G'$  is constant throughout the whole height of the tower and, because only a small proportion of the total supply of water is normally evaporated (1–5 per cent), the liquid rate per unit area  $L'$  can be taken as constant. The temperature, enthalpy, and humidity will be denoted by the symbols  $\theta$ ,  $H$ , and  $\mathcal{H}$  respectively, suffixes  $G$ ,  $L$ , 1, 2, and  $f$  being used to denote conditions in the gas and liquid, at the bottom and top of the column, and of the air in contact with the water.

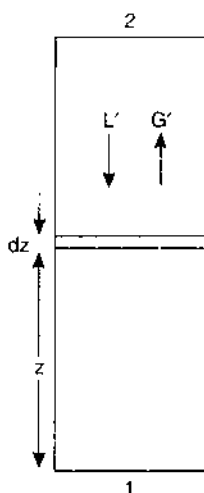


Figure 13.15. Flow in water-cooling tower

The five basic equations for an incremental height of column,  $dz$ , are:

$$(1) \text{ Water balance: } dL' = G' d\mathcal{H} \quad (13.33)$$

$$(2) \text{ Enthalpy balance: } G' dH_G = L' dH_L \quad (13.34)$$

since only a small proportion of the liquid is evaporated.

$$\text{Now: } H_G = s(\theta_G - \theta_0) + \lambda \mathcal{H} \quad (13.35)$$

$$\text{and: } H_L = C_L(\theta_L - \theta_0) \quad (13.36)$$

$$\text{Thus: } G' dH_G = L' C_L d\theta_L \quad (13.37)$$

$$\text{and: } dH_G = s d\theta_G + \lambda d\mathcal{H} \quad (13.38)$$

Integration of this expression over the whole height of the column, on the assumption that the physical properties of the materials do not change appreciably, gives:

$$G'(H_{G2} - H_{G1}) = L' C_L (\theta_{L2} - \theta_{L1}) \quad (13.39)$$

(3) Heat transfer from the body of the liquid to the interface:

$$h_L a dz (\theta_L - \theta_f) = L' C_L d\theta_L \quad (13.40)$$

where  $h_L$  is the heat transfer coefficient in the liquid phase and  $a$  is the interfacial area per unit volume of column. It will be assumed that the area for heat transfer is equal to that available for mass transfer, though it may be somewhat greater if the packing is not completely wetted.

Rearranging equation 13.40:

$$\frac{d\theta_L}{(\theta_L - \theta_f)} = \frac{h_L a}{L' C_L} dz \quad (13.41)$$

(4) Heat transfer from the interface to the bulk of the gas:

$$h_G a dz (\theta_f - \theta_G) = G' s d\theta_G \quad (13.42)$$

where  $h_G$  is the heat transfer coefficient in the gas phase.

$$\text{Rearranging: } \frac{d\theta_G}{(\theta_f - \theta_G)} = \frac{h_G a}{G' s} dz \quad (13.43)$$

(5) Mass transfer from the interface to the gas:

$$h_D \rho a dz (\mathcal{H}_f - \mathcal{H}) = G' d\mathcal{H} \quad (13.44)$$

where  $h_D$  is the mass transfer coefficient for the gas and  $\rho$  is the mean density of the air (see equation 13.6).

$$\text{Rearranging: } \frac{d\mathcal{H}}{(\mathcal{H}_f - \mathcal{H})} = \frac{h_D \rho a}{G'} dz \quad (13.45)$$

These equations cannot be integrated directly since the conditions at the interface are not necessarily constant; nor can they be expressed directly in terms of the corresponding property in the bulk of the gas or liquid.

If the Lewis relation (equation 13.11) is applied, it is possible to obtain workable equations in terms of enthalpy instead of temperature and humidity. Thus, writing  $h_L$  as  $h_D \rho_s$ , from equation 13.42:

$$G' s d\theta_G = h_D \rho_a dz (s\theta_f - s\theta_G) \quad (13.46)$$

and from equation 13.44:

$$G' \lambda d\mathcal{H} = h_D \rho_a dz (\lambda \mathcal{H}_f - \lambda \mathcal{H}) \quad (13.47)$$

Adding these two equations gives:

$$\begin{aligned} G' (s d\theta_G + \lambda d\mathcal{H}) &= h_D \rho_a dz [(s\theta_f + \lambda \mathcal{H}_f) - (s\theta_G + \lambda \mathcal{H})] \\ G' dH_G &= h_D \rho_a dz (H_f - H_G) \quad (\text{from equation 13.35}) \end{aligned} \quad (13.48)$$

$$\text{or:} \quad \frac{dH_G}{(H_f - H_G)} = \frac{h_D \rho_a}{G'} dz \quad (13.49)$$

The use of an enthalpy driving force, as in equation 13.48, was first suggested by MERKEL<sup>(13)</sup>, and the following development of the treatment was proposed by MICKLEY<sup>(12)</sup>.

Combining of equations 13.37, 13.40, and 13.48 gives:

$$\frac{(H_G - H_f)}{(\theta_L - \theta_f)} = - \frac{h_L}{h_D \rho} \quad (13.50)$$

From equations 13.46 and 13.48:

$$\frac{(H_G - H_f)}{(\theta_G - \theta_f)} = \frac{dH_G}{d\theta_G} \quad (13.51)$$

and from equations 13.46 and 13.44:

$$\frac{(\mathcal{H} - \mathcal{H}_f)}{(\theta_G - \theta_f)} = \frac{d\mathcal{H}}{d\theta_G} \quad (13.52)$$

These equations are now employed in the determination of the required height of a cooling tower for a given duty. The method consists of the graphical evaluation of the relation between the enthalpy of the body of gas and the enthalpy of the gas at the interface with the liquid. The required height of the tower is then obtained by integration of equation 13.49.

It is supposed that water is to be cooled at a mass rate  $L'$  per unit area from a temperature  $\theta_{L,2}$  to  $\theta_{L,1}$ . The air will be assumed to have a temperature  $\theta_{G,1}$ , a humidity  $\mathcal{H}_1$ , and an enthalpy  $H_{G,1}$  (which can be calculated from the temperature and humidity), at the inlet point at the bottom of the tower, and its mass flow per unit area will be taken as  $G'$ . The change in the condition of the liquid and gas phases will now be followed on an enthalpy-temperature diagram (Figure 13.16). The enthalpy-temperature curve  $PQ$  for saturated air is plotted either using calculated data or from the humidity chart (Figure 13.4). The region below this line relates to unsaturated air and the region above it to supersaturated air. If it is assumed that the air in contact with the liquid surface



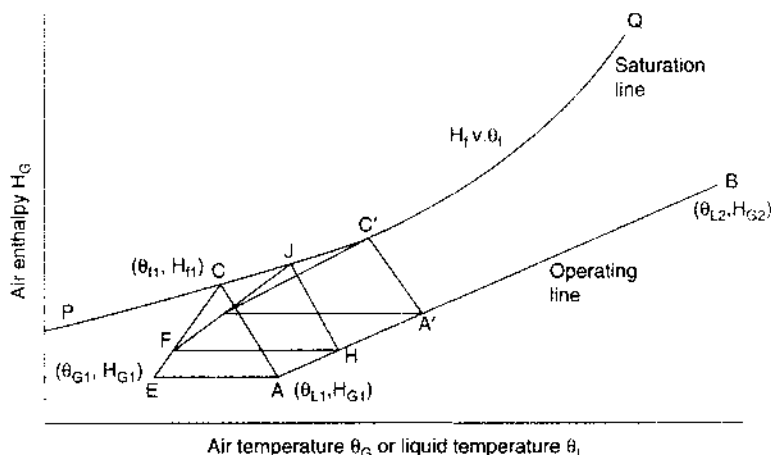


Figure 13.16. Construction for determining the height of water-cooling tower

is saturated with water vapour, this curve represents the relation between air enthalpy  $H_f$  and temperature  $\theta_f$  at the interface.

The curve connecting air enthalpy and water temperature is now drawn using equation 13.39. This is known as the operating line and is a straight line of slope  $(L'C_L/G')$ , passing through the points  $A(\theta_{L1}, H_{G1})$  and  $B(\theta_{L2}, H_{G2})$ . Since  $\theta_{L1}, H_{G1}$  are specified, the procedure is to draw a line through  $(\theta_{L1}, H_{G1})$  of slope  $L'C_L/G'$  and to produce it to a point whose abscissa is equal to  $\theta_{L2}$ . This point  $B$  then corresponds to conditions at the top of the tower and the ordinate gives the enthalpy of the air leaving the column.

Equation 13.50 gives the relation between liquid temperature, air enthalpy, and conditions at the interface, for any position in the tower, and is represented by a family of straight lines of slope  $-(h_L/h_D\rho)$ . The line for the bottom of the column passes through the point  $A(\theta_{L1}, H_{G1})$  and cuts the enthalpy-temperature curve for saturated air at the point  $C$ , representing conditions at the interface. The difference in ordinates of points  $A$  and  $C$  is the difference in the enthalpy of the air at the interface and that of the bulk air at the bottom of the column.

Similarly, line  $A'C'$ , parallel to  $AC$ , enables the difference in the enthalpies of the bulk air and the air at the interface to be determined at some other point in the column. The procedure can be repeated for a number of points and the value of  $(H_f - H_G)$  obtained as a function of  $H_G$  for the whole tower.

$$\text{Now:} \quad \frac{dH_G}{(H_f - H_G)} = \frac{h_D a \rho}{G'} dz \quad (\text{equation 13.49})$$

On integration:

$$z = \int_1^2 dz = \frac{G'}{h_D a \rho} \int_1^2 \frac{dH_G}{(H_f - H_G)} \quad (13.53)$$

assuming  $h_D$  to remain approximately constant.

Since  $(H_f - H_G)$  is now known as a function of  $H_G$ ,  $1/(H_f - H_G)$  can be plotted against  $H_G$  and the integral evaluated between the required limits. The height of the tower is thus determined.

The integral in equation 13.53 cannot be evaluated by taking a logarithmic mean driving force because the saturation line  $PQ$  is far from linear. CAREY and WILLIAMSON<sup>(14)</sup> have given a useful approximate method of evaluating the integral. They assume that the enthalpy difference  $(H_f - H_G) = \Delta H$  varies in a parabolic manner. The three fixed points taken to define the parabola are at the bottom and top of the column ( $\Delta H_1$  and  $\Delta H_2$  respectively) and  $\Delta H_m$ , the value at the mean water temperature in the column. The effective mean driving force is  $f \Delta H_m$ , where  $f$  is a factor for converting the driving force at the mean water temperature to the effective value. In Figure 13.17,  $(\Delta H_m/\Delta H_1)$  is plotted against  $(\Delta H_m/\Delta H_2)$  and contours representing constant values of  $f$  are included.

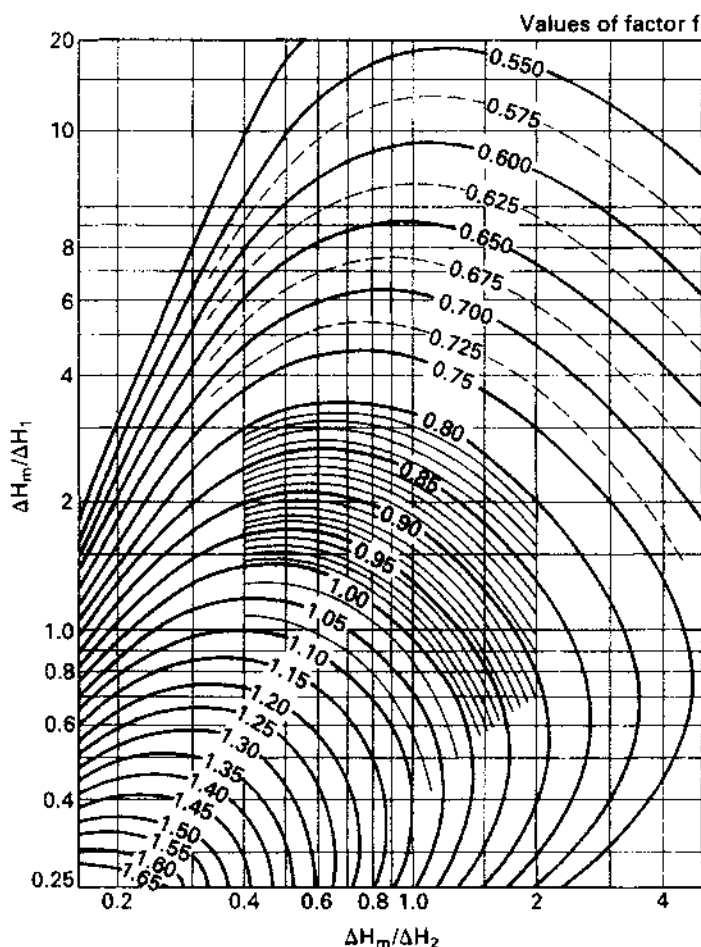


Figure 13.17. Correction factor  $f$  for obtaining the mean effective driving force in column

Using the mean driving force, integration of equation 13.53 gives:

$$\frac{(H_{G2} - H_{G1})}{f \Delta H_m} = \frac{h_D a \rho}{G'} z \quad (13.54)$$

or:

$$z = \frac{G' (H_{G2} - H_{G1})}{h_D a \rho f \Delta H_m}$$

### 13.6.4. Change in air condition

The change in the humidity and temperature of the air is now obtained. The enthalpy and temperature of the air are known only at the bottom of the tower, where fresh air is admitted. Here the condition of the air may be represented by a point *E* with coordinates  $(H_{G1}, \theta_{G1})$ . Thus the line *AE* (Figure 13.16) is parallel to the temperature axis.

Since:

$$\frac{H_G - H_f}{\theta_G - \theta_f} = \frac{dH_G}{d\theta_G} \quad (13.51)$$

the slope of the line *EC* is  $(dH_G/d\theta_G)$  and represents the rate of change of air enthalpy with air temperature at the bottom of the column. If the gradient  $(dH_G/d\theta_G)$  is taken as constant over a small section, the point *F*, on *EC*, will represent the condition of the gas at a small distance from the bottom. The corresponding liquid temperature is found by drawing through *F* a line parallel to the temperature axis. This cuts the operating line at some point *H*, which indicates the liquid temperature. The corresponding value of the temperature and enthalpy of the gas at the interface is then obtained by drawing a line through *H*, parallel to *AC*. This line then cuts the curve for saturated air at a point *J*, which represents the conditions of the gas at the interface. The rate of change of enthalpy with temperature for the gas is then given by the slope of the line *FJ*. Again, this slope can be considered to remain constant over a small height of the column, and the condition of the gas is thus determined for the next point in the tower. The procedure is then repeated until the curve representing the condition of the gas has been extended to a point whose ordinate is equal to the enthalpy of the gas at the top of the column. This point is obtained by drawing a straight line through *B*, parallel to the temperature axis. The final point on the line then represents the condition of the air which leaves the top of the water-cooling tower.

The size of the individual increments of height which are considered must be decided for the particular problem under consideration and will depend, primarily, on the rate of change of the gradient  $(dH_G/d\theta_G)$ . It should be noted that, for the gas to remain unsaturated throughout the whole of the tower, the line representing the condition of the gas must be below the curve for saturated gas. If at any point in the column, the air has become saturated, it is liable to become supersaturated as it passes further up the column and comes into contact with hotter liquid. It is difficult to define precisely what happens beyond this point as partial condensation may occur, giving rise to a mist. Under these conditions the preceding equations will no longer be applicable. However, an approximate solution is obtained by assuming that once the air stream becomes saturated it remains so during its subsequent contact with the water through the column.

### 13.6.5. Temperature and humidity gradients in a water cooling tower

In a water cooling tower, the temperature profiles depend on whether the air is cooler or hotter than the surface of the water. Near the top, hot water makes contact with the exit air which is at a lower temperature, and sensible heat is therefore transferred both from the water to the interface and from the interface to the air. The air in contact with the water is saturated at the interface temperature and humidity therefore falls from the interface to the air. Evaporation followed by mass transfer of water vapour therefore takes place and latent heat is carried away from the interface in the vapour. The sensible heat removed from the water is then equal to the sum of the latent and sensible heats transferred to the air. Temperature and humidity gradients are then as shown in Figure 13.18*a*.

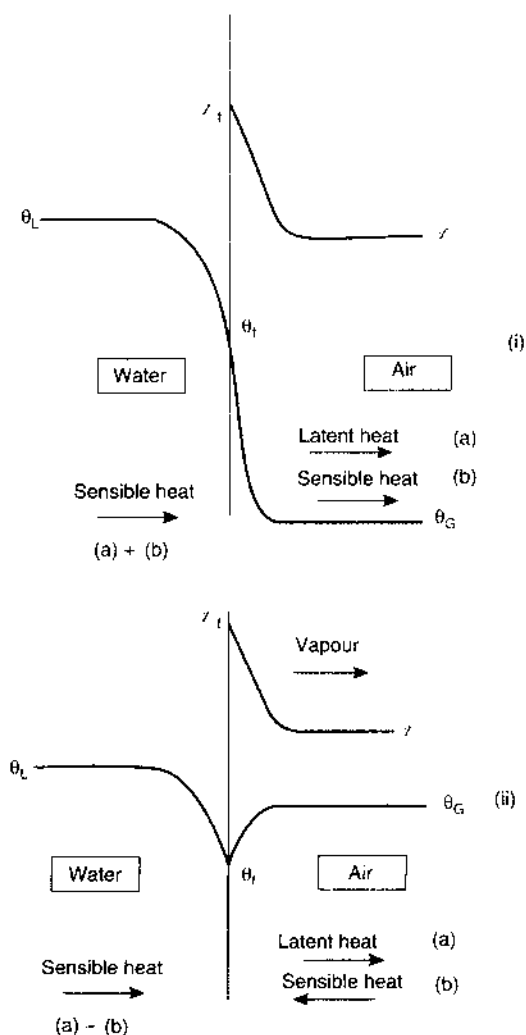


Figure 13.18. Temperature and humidity gradients in a water cooling tower (i) upper sections of tower (ii) bottom of tower

If the tower is sufficiently tall, the interface temperature can fall below the dry bulb temperature of the air (but not below its wet bulb temperature), and sensible heat will then be transferred from both the air and the water to the interface. The corresponding temperature and humidity profiles are given in Figure 13.18*b*. In this part of the tower, therefore, the sensible heat removed from the water will be that transferred as latent heat *less* the sensible heat transferred from the air.

### 13.6.6. Evaluation of heat and mass transfer coefficients

In general, coefficients of heat and mass transfer in the gas phase and the heat transfer coefficient for the liquid phase are not known. They may be determined, however, by carrying out tests in the laboratory or pilot scale using the same packing. If, for the air-water system, a small column is operated at steady water and air rates and the temperature of the water at the top and bottom and the initial and final temperatures and humidities of the air stream are noted, the operating line for the system is obtained. Assuming a value of the ratio  $-(h_L/h_{D\rho})$ , for the slope of the tie-lines *AC*, etc., the graphical construction is carried out, starting with the conditions at the bottom of the tower. The condition of the gas at the top of the tower is thus calculated and compared with the measured value. If the difference is significant, another value of  $-(h_L/h_{D\rho})$  is assumed and the procedure repeated. Now that the slope of the tie line is known, the value of the integral of  $dH_G/(H_f - H_G)$  over the whole column can be calculated. Since the height of the column is known, the product  $h_{Da}$  is found by solution of equation 13.49.  $h_{Ga}$  may then be calculated using the Lewis relation. The values of the three transfer coefficients are therefore obtained at any given flow rates from a single experimental run. The effect of liquid and gas rate may be found by means of a series of similar experiments.

Several workers have measured heat and mass transfer coefficients in water-cooling towers and in humidifying towers. THOMAS and HOUSTON<sup>(15)</sup>, using a tower 2 m high and 0.3 m square in cross-section, fitted with wooden slats, give the following equations for heat and mass transfer coefficients for packed heights greater than 75 mm:

$$h_{Ga} = 3.0L'^{0.26}G'^{0.72} \quad (13.55)$$

$$h_{La} = 1.04 \times 10^4 L'^{0.51} G'^{1.00} \quad (13.56)$$

$$h_{Da} = 2.95L'^{0.26}G'^{0.72} \quad (13.57)$$

In these equations,  $L'$  and  $G'$  are expressed in  $\text{kg/m}^2\text{s}$ ,  $s$  in  $\text{J/kg K}$ ,  $h_{Ga}$  and  $h_{La}$  in  $\text{W/m}^2\text{K}$ , and  $h_{Da}$  in  $\text{s}^{-1}$ . A comparison of the gas and liquid film coefficients may then be made for a number of gas and liquid rates. Taking the humid heat  $s$  as  $1.17 \times 10^3 \text{ J/kg K}$ :

	$L' = G' = 0.5 \text{ kg/m}^2\text{s}$	$L' = G' = 1.0 \text{ kg/m}^2\text{s}$	$L' = G' = 2.0 \text{ kg/m}^2\text{s}$
$h_{Ga}$	1780	3510	6915
$h_{La}$	3650	10,400	29,600
$h_{La}/h_{Ga}$	2.05	2.96	4.28

CRIBB<sup>(16)</sup> quotes values of the ratio  $h_L/h_G$  ranging from 2.4 to 8.5.

It is seen that the liquid film coefficient is generally considerably higher than the gas film coefficient, but that it is not always safe to ignore the resistance to transfer in the liquid phase.

LOWE and CHRISTIE<sup>(17)</sup> used a 1.3 m square experimental column fitted with a number of different types of packing and measured heat and mass transfer coefficients and pressure drops. They showed that in most cases:

$$h_{pa} \propto L^{1-n} G^n \quad (13.58)$$

The index  $n$  was found to vary from about 0.4 to 0.8 according to the type of packing. It will be noted that when  $n \approx 0.75$ , there is close agreement with the results given by equation 13.57.

The heat-transfer coefficient for the liquid is often large compared with that for the gas phase. As a first approximation, therefore, it can be assumed that the whole of the resistance to heat transfer lies within the gas phase and that the temperature at the water-air interface is equal to the temperature of the bulk of the liquid. Thus, everywhere in the tower,  $\theta_f = \theta_L$ . This simplifies the calculations, since the lines  $AC$ ,  $HJ$ , and so on, have a slope of  $-\infty$ , that is, they become parallel to the enthalpy axis.

Some workers have attempted to base the design of humidifiers on the overall heat transfer coefficient between the liquid and gas phases. This treatment is not satisfactory since the quantities of heat transferred through the liquid and through the gas are not the same, as some of the heat is utilised in effecting evaporation at the interface. In fact, at the bottom of a tall tower, the transfer of heat in both the liquid and the gas phases may be towards the interface, as already indicated. A further objection to the use of overall coefficients is that the Lewis relation may be applied only to the heat and mass transfer coefficients in the gas phase.

In the design of commercial units, nomographs<sup>(18,19)</sup> are available which give a performance characteristic ( $KaV/L'$ ), where  $K$  is a mass transfer coefficient ( $\text{kg water/m}^2\text{s}$ ) and  $V$  is the active cooling volume ( $\text{m}^3/\text{m}^2$  plan area), as a function of  $\theta$ ,  $\theta_w$  and  $(L'/G')$ . For a given duty ( $KaV/L'$ ) is calculated from:

$$\frac{KaV}{C_L L'} = \int_{\theta_1}^{\theta_2} \frac{d\theta}{(H_f - H_G)} \quad (13.59)$$

and then a suitable tower with this value of ( $KaV/L'$ ) is sought from performance curves<sup>(20,21)</sup>. In normal applications the performance characteristic varies between 0.5–2.5.

### Example 13.9

Water is to be cooled from 328 to 293 K by means of a countercurrent air stream entering at 293 K with a relative humidity of 20 per cent. The flow of air is  $0.68 \text{ m}^3/\text{m}^2\text{s}$  and the water throughput is  $0.26 \text{ kg/m}^2\text{s}$ . The whole of the resistance to heat and mass transfer may be assumed to be in the gas phase and the product, ( $h_{pa}$ ), may be taken as  $0.2 (\text{m/s})(\text{m}^2/\text{m}^3)$ , that is  $0.2 \text{ s}^{-1}$ .

What is the required height of packing and the condition of the exit air stream?

### Solution

Assuming the latent heat of water at 273 K = 2495 kJ/kg  
 specific heat of air = 1.003 kJ/kg K  
 and specific heat of water vapour = 2.006 kJ/kg K

the enthalpy of the inlet air stream,

$$H_{G1} = 1.003(293 - 273) + \mathcal{H}[2495 + 2.006(293 - 273)]$$

From Figure 13.4:

at  $\theta = 293$  K and 20 per cent RH,  $\mathcal{H} = 0.003$  kg/kg, and hence

$$\begin{aligned} H_{G1} &= (1.003 \times 20) + 0.003[2495 + (2.006 \times 20)] \\ &= 27.67 \text{ kJ/kg} \end{aligned}$$

In the inlet air, water vapour = 0.003 kg/kg dry air

$$\text{or: } \frac{(0.003/18)}{(1/29)} = 0.005 \text{ kmol/kmol dry air}$$

$$\text{Thus flow of dry air} = (1 - 0.005)0.68 = 0.677 \text{ m}^3/\text{m}^2\text{s}$$

$$\text{Density of air at 293 K} = \left(\frac{29}{22.4}\right) \left(\frac{273}{293}\right) = 1.206 \text{ kg/m}^3$$

$$\text{and mass flow of dry air} = (1.206 \times 0.677) = 0.817 \text{ kg/m}^2\text{s}$$

$$\text{Slope of operating line: } (L' C_L / G') = \frac{(0.26 \times 4.18)}{0.817} = 1.33$$

The coordinates of the bottom of the operating line are:

$$\theta_{L1} = 293 \text{ K, } H_{G1} = 27.67 \text{ kJ/kg}$$

Hence on an enthalpy-temperature diagram, the operating line of slope 1.33 is drawn through the point  $(293, 27.67) = (\theta_{L1}, H_{G1})$ .

The top point of the operating line is given by  $\theta_{L2} = 328$  K, and  $H_{G2}$  is found to be 76.5 kJ/kg (Figure 13.19).

From Figures 13.4 and 13.5 the curve representing the enthalpy of saturated air as a function of temperature is obtained and drawn in. Alternatively, this plot may be calculated from:

$$H_F = C_a(\theta_f - 273) + \mathcal{H}_0[C_w(\theta_f - 273) + \lambda] \text{ kJ/kg}$$

The curve represents the relation between enthalpy and temperature at the interface, that is  $H_f$  as a function of  $\theta_f$ .

It now remains to evaluate the integral  $\int dH_G/(H_f - H_G)$  between the limits,  $H_{G1} = 27.7$  kJ/kg and  $H_{G2} = 76.5$  kJ/kg. Various values of  $H_G$  between these limits are selected and the value of  $\theta$  obtained from the operating line. At this value of  $\theta$ , now  $\theta_f$ , the corresponding value of  $H_f$  is obtained from the curve for saturated air. The working is as follows:

$H_G$	$\theta = \theta_f$	$H_f$	$(H_f - H_G)$	$1/(H_f - H_G)$
27.7	293	57.7	30	0.0330
30	294.5	65	35	0.0285
40	302	98	58	0.0172
50	309	137	87	0.0114
60	316	190	130	0.0076
70	323	265	195	0.0051
76.5	328	355	279	0.0035

A plot of  $1/(H_f - H_G)$  and  $H_G$  is now made as shown in Figure 13.20 from which the area under the curve = 0.65. This value may be checked using the approximate solution of CAREY and WILLIAMSON<sup>(14)</sup>.

At the bottom of the column:

$$H_{G1} = 27.7 \text{ kJ/kg, } H_{f1} = 57.7 \text{ kJ/kg, } \therefore \Delta H_1 = 30 \text{ kJ/kg}$$

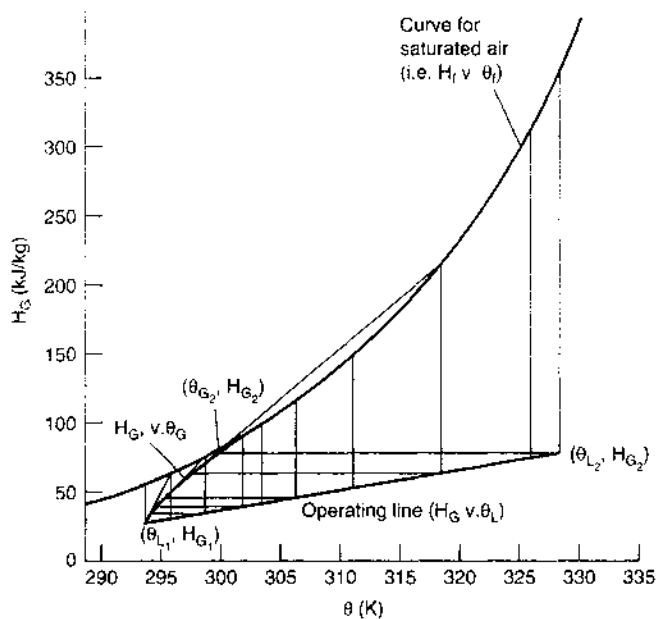
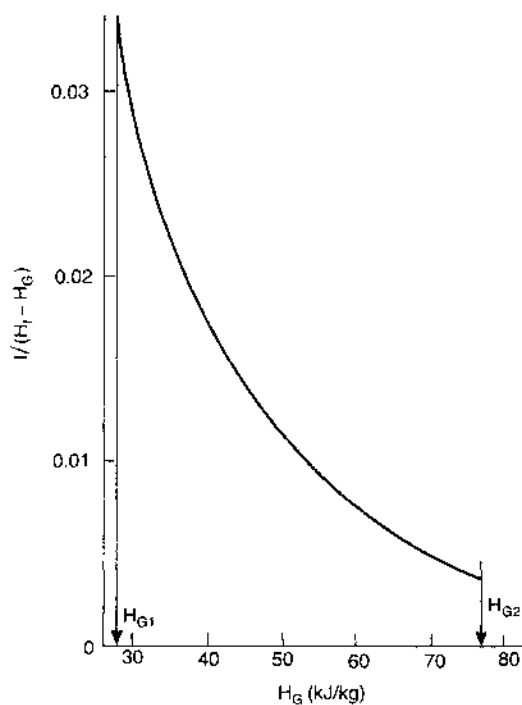


Figure 13.19. Calculation of the height of a water-cooling tower

Figure 13.20. Evaluation of the integral of  $dH_G / (H_i - H_G)$



At the top of the column:

$$H_{G2} = 76.5 \text{ kJ/kg}, \quad H_{I2} = 355 \text{ kJ/kg} \quad \therefore \Delta H_2 = 279 \text{ kJ/kg}$$

At the mean water temperature of  $0.5(328 + 293) = 310.5 \text{ K}$ :

$$H_{Gm} = 52 \text{ kJ/kg}, \quad H_f = 145 \text{ kJ/kg} \quad \therefore \Delta H_m = 93 \text{ kJ/kg}$$

$$\frac{\Delta H_m}{\Delta H_1} = 3.10, \quad \frac{\Delta H_m}{\Delta H_2} = 0.333,$$

and from Figure 13.16:  $f = 0.79$

$$\text{Thus:} \quad \frac{(H_{G2} - H_{G1})}{f \Delta H_m} = \frac{(76.5 - 27.7)}{(0.79 \times 93)} = 0.66$$

which agrees well with the value (0.65) obtained by graphical integration.

Thus, in equation 13.53:

$$\begin{aligned} \text{height of packing, } z &= \int_{H_{G1}}^{H_{G2}} \frac{dH_G}{(H_f - H_G)} \frac{G'}{h_{p,a} \rho} \\ &= \frac{(0.65 \times 0.817)}{(0.2 \times 1.206)} \\ &= \underline{\underline{2.20 \text{ m}}} \end{aligned}$$

Assuming that the resistance to mass transfer lies entirely within the gas phase, the lines connecting  $\theta_L$  and  $\theta_f$  are parallel with the enthalpy axis.

In Figure 13.18 a plot of  $H_G$  and  $\theta_G$  is obtained using the construction given in Section 13.6.4 and shown in Figure 13.15. From this curve, the value of  $\theta_{G2}$  corresponding to  $H_{G2} = 76.5 \text{ kJ/kg}$  is  $300 \text{ K}$ . From Figure 13.5, under these conditions, the exit air has a humidity of  $0.019 \text{ kg/kg}$  which from Figure 13.4 corresponds to a relative humidity of 83 per cent.

### 13.6.7. Humidifying towers

If the main function of the tower is to produce a stream of humidified air, the final temperature of the liquid will not be specified, and the humidity of the gas leaving the top of the tower will be given instead. It is therefore not possible to fix any point on the operating line, though its slope can be calculated from the liquid and gas rates. In designing a humidifier, therefore, it is necessary to calculate the temperature and enthalpy, and hence the humidity, of the gas leaving the tower for a number of assumed water outlet temperatures and thereby determine the outlet water temperature resulting in the air leaving the tower with the required humidity. The operating line for this water-outlet temperature is then used in the calculation of the height of the tower required to effect this degree of humidification. The calculation of the dimensions of a humidifier is therefore rather more tedious than that for the water-cooling tower.

In a humidifier in which the make-up liquid is only a small proportion of the total liquid circulating, its temperature approaches the adiabatic saturation temperature  $\theta_s$ , and remains constant, so that there is no temperature gradient in the liquid. The gas in contact with the liquid surface is approximately saturated and has a humidity  $\mathcal{H}_s$ .

$$\text{Thus:} \quad d\theta_L = 0$$

$$\text{and:} \quad \theta_{L1} = \theta_{L2} = \theta_L = \theta_f = \theta_s$$

$$\text{Hence:} \quad -G' s d\theta_G = h_G a dz (\theta_G - \theta_s) \quad (\text{from equation 13.42})$$

$$\text{and:} \quad -G' d\mathcal{H} = h_D \rho a dz (\mathcal{H} - \mathcal{H}_s) \quad (\text{from equation 13.44})$$

Integration of these equations gives:

$$\ln \frac{(\theta_{G1} - \theta_s)}{(\theta_{G2} - \theta_s)} = \frac{h_G a}{G' s} z \quad (13.60)$$

and:

$$\ln \frac{(\mathcal{H}_s - \mathcal{H}_1)}{(\mathcal{H}_s - \mathcal{H}_2)} = \frac{h_D a p}{G' s} z \quad (13.61)$$

assuming  $h_G$ ,  $h_D$ , and  $s$  remain approximately constant.

From these equations the temperature  $\theta_{G2}$  and the humidity  $\mathcal{H}_2$  of the gas leaving the humidifier may be calculated in terms of the height of the tower. Rearrangement of equation 13.61 gives:

$$\ln \left( 1 + \frac{\mathcal{H}_1 - \mathcal{H}_2}{\mathcal{H}_s - \mathcal{H}_1} \right) = - \frac{h_D a p}{G' s} z$$

or:

$$\frac{(\mathcal{H}_2 - \mathcal{H}_1)}{(\mathcal{H}_s - \mathcal{H}_1)} = 1 - e^{-h_D a p z / G' s} \quad (13.62)$$

Thus the ratio of the actual increase in humidity produced in the saturator to the maximum possible increase in humidity (that is, the production of saturated gas) is equal to  $(1 - e^{-h_D a p z / G' s})$ , and complete saturation of the gas is reached exponentially. A similar relation exists for the change in the temperature of the gas stream:

$$\frac{(\theta_{G1} - \theta_{G2})}{(\theta_{G1} - \theta_s)} = 1 - e^{-h_G a z / G' s} \quad (13.63)$$

Further, the relation between the temperature and the humidity of the gas at any stage in the adiabatic humidifier is given by:

$$\frac{d\mathcal{H}}{d\theta_G} = \frac{(\mathcal{H} - \mathcal{H}_s)}{(\theta_G - \theta_s)} \quad (\text{from equation 13.52})$$

On integration:

$$\ln \frac{(\mathcal{H}_s - \mathcal{H}_2)}{(\mathcal{H}_s - \mathcal{H}_1)} = \ln \frac{(\theta_{G2} - \theta_s)}{(\theta_{G1} - \theta_s)} \quad (13.64)$$

or:

$$\frac{(\mathcal{H}_s - \mathcal{H}_2)}{(\mathcal{H}_s - \mathcal{H}_1)} = \frac{(\theta_{G2} - \theta_s)}{(\theta_{G1} - \theta_s)} \quad (13.65)$$

### 13.7. SYSTEMS OTHER THAN AIR-WATER

Calculations involving to systems where the Lewis relation is not applicable are very much more complicated because the adiabatic saturation temperature and the wet-bulb temperature do not coincide. Thus the significance of the adiabatic cooling lines on the psychrometric chart is very much restricted. They no longer represent the changes which take place in a gas as it is humidified by contact with liquid initially at the adiabatic saturation temperature of the gas, but simply give the compositions of all gases with the same adiabatic saturation temperature.

Calculation of the change in the condition of the liquid and the gas in a humidification tower is rendered more difficult since equation 13.49, which was derived for the air-water system, is no longer applicable. LEWIS and WHITE<sup>(22)</sup> have developed a method of

calculation based on the use of a *modified enthalpy* in place of the true enthalpy of the system.

For the air-water system, from equation 13.11:

$$h_G = h_D \rho s \quad (13.66)$$

This relationship applies quite closely for the conditions normally encountered in practice. For other systems, the relation between the heat and mass transfer coefficients in the gas phase is given by:

$$h_G = b h_D \rho s \quad (13.67)$$

where  $b$  is approximately constant and generally has a value greater than unity.

For these systems, equation 13.46 becomes:

$$G' s d\theta_G = b h_D \rho a dz (s\theta_f - s\theta_G) \quad (13.68)$$

Adding equations 13.68 and 13.47 to obtain the relationship corresponding to equation 13.48 gives:

$$G' (s d\theta_G + \lambda d\mathcal{H}) = h_D \rho a dz [(bs\theta_f + \lambda \mathcal{H}_f) - (bs\theta_G + \lambda \mathcal{H})] \quad (13.69)$$

Lewis and White use a *modified latent heat of vaporisation*  $\lambda'$  defined by:

$$b = \frac{\lambda}{\lambda'} \quad (13.70)$$

and a *modified enthalpy* per unit mass of dry gas defined by:

$$H'_G = s(\theta_G - \theta_0) + \lambda' \mathcal{H} \quad (13.71)$$

Substituting in equation 13.67; from equations 13.38, 13.70, and 13.71:

$$G' dH_G = b h_D \rho a dz (H'_f - H'_G) \quad (13.72)$$

and:

$$\frac{dH_G}{(H'_f - H'_G)} = \frac{b h_D \rho a}{G'} dz \quad (13.73)$$

Combining equations 13.37, 13.40, and 13.72:

$$\frac{(H'_G - H'_f)}{(\theta_L - \theta_f)} = - \frac{h_L}{h_D \rho b} \quad (\text{cf. equation 13.50}) \quad (13.74)$$

From equations 13.66 and 13.72:

$$\frac{(H'_G - H'_f)}{(\theta_G - \theta_f)} = \frac{dH_G}{d\theta_G} \quad (\text{cf. equation 13.51}) \quad (13.75)$$

From equations 13.44 and 13.67:

$$\frac{(\mathcal{H} - \mathcal{H}_f)}{(\theta_G - \theta_f)} = b \frac{d\mathcal{H}}{d\theta_G} \quad (\text{cf. equation 13.52}) \quad (13.76)$$

The calculation of conditions within a countercurrent column operating with a system other than air-water is carried out in a similar manner to that already described by

applying equations 13.73, 13.74, and 13.75 in conjunction with equation 3.39:

$$G'(H_{G2} - H_{G1}) = L'C_L(\theta_{L2} - \theta_{L1}) \quad (\text{equation 13.39})$$

On an enthalpy-temperature diagram (Figure 13.20) the enthalpy of saturated gas is plotted against its temperature. If equilibrium between the liquid and gas exists at the interface, this curve  $PQ$  represents the relation between gas enthalpy and temperature at the interface ( $H_f$  v.  $\theta_f$ ). The modified enthalpy of saturated gas is then plotted against temperature (curve  $RS$ ) to give the relation between  $H'_f$  and  $\theta_f$ . Since  $b$  is greater than unity,  $RS$  will lie below  $PQ$ . By combining equations 13.35, 13.70, and 13.72,  $H'_G$  is obtained in terms of  $H_G$ .

$$H'_G = \frac{1}{b}[H_G + (b-1)s(\theta_G - \theta_0)] \quad (13.77)$$

$H'_G$  may be conveniently plotted against  $H_G$  for a number of constant temperatures. If  $b$  and  $s$  are constant, a series of straight lines is obtained. The operating line  $AB$  given by equation 13.39 is drawn in Figure 13.21. Point  $A$  has coordinates  $(\theta_{L1}, H_{G1})$  corresponding to the bottom of the column. Point  $a$  has coordinates  $(\theta_{f1}, H'_{G1})$ ,  $H'_{G1}$  being obtained from equation 13.77.

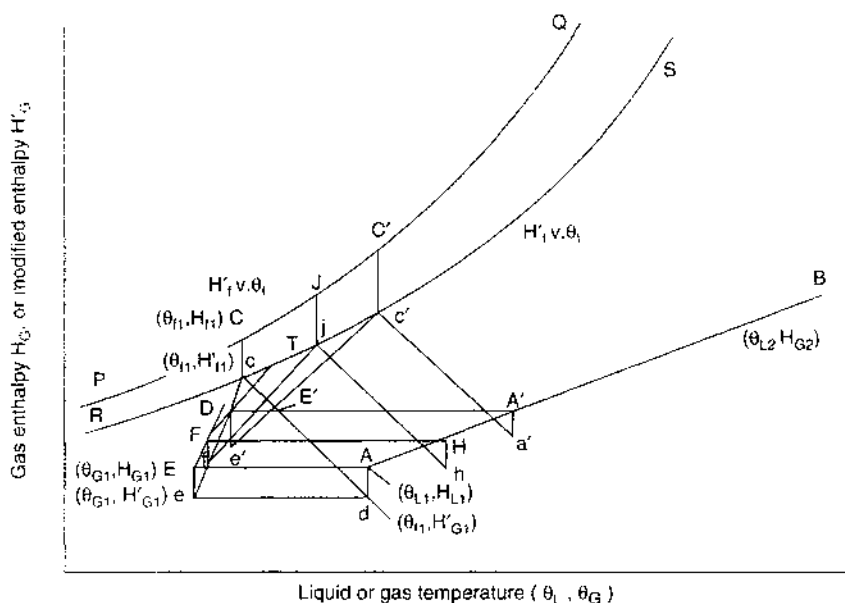


Figure 13.21. Construction for height of a column for vapour other than water

From equation 13.72, a line through  $a$ , of slope  $-(h_L/h_D\rho b)$ , will intersect curve  $RS$  at  $c$ ,  $(\theta_{f1}, H'_{f1})$  to give the interface conditions at the bottom of the column. The corresponding air enthalpy is given by  $C$ ,  $(\theta_{f1}, H_{f1})$ . The difference between the ordinates of  $c$  and  $a$  then gives the driving force in terms of modified enthalpy at the bottom of the column  $(H'_{f1} - H'_{G1})$ . A similar construction at other points, such as  $A'$ , enables the

driving force to be calculated at any other point. Hence  $(H'_f - H'_G)$  is obtained as a function of  $H_G$  throughout the column. The height of column corresponding to a given change in air enthalpy can be obtained from equation 13.71 since the left-hand side can now be evaluated.

Thus:

$$\int_{H_{G1}}^{H_{G2}} \frac{dH_G}{(H'_f - H'_G)} = \frac{bh_D a \rho}{G'} z \quad (13.78)$$

The change in the condition of the gas stream is obtained as follows:  $E$ , with coordinates  $(\theta_{G1}, H_{G1})$ , represents the condition of the inlet gas. The modified enthalpy of this gas is given by  $e(\theta_{G1}, H'_{G1})$ . From equation 13.75 it is seen that  $ec$  gives the rate of change of gas enthalpy with temperature  $(dH_G/d\theta_G)$  at the bottom of the column. Thus  $ED$ , parallel to  $ec$ , describes the way in which gas enthalpy changes at the bottom of the column. At some arbitrary small distance from the bottom,  $F$  represents the condition of the gas and  $H$  gives the corresponding liquid temperature. In exactly the same way the next small change is obtained by drawing a line  $hj$  through  $h$  parallel to  $ac$ . The slope of  $ff$  gives the new value of  $(dH_G/d\theta_G)$  and therefore the gas condition at a higher point in the column is obtained by drawing  $FT$  parallel to  $ff$ . In this way the change in the condition of the gas through the column can be followed by continuing the procedure until the gas enthalpy reaches the value  $H_{G2}$  corresponding to the top of the column.

A detailed description of the method of construction of psychrometric charts is given by SHALLCROSS and LOW<sup>(23)</sup>, who illustrate their method by producing charts for three systems: air-water, air-benzene and air-toluene at pressures of 1 and 2 bar.

### Example 13.10

In a countercurrent packed column, n-butanol flows down at a rate of  $0.25 \text{ kg/m}^2 \text{ s}$  and is cooled from 330 to 295 K. Air at 290 K, initially free of n-butanol vapour, is passed up the column at the rate of  $0.7 \text{ m}^3/\text{m}^2 \text{ s}$ . Calculate the required height of tower and the condition of the exit air.

Data:

Mass transfer coefficient per unit volume,  $h_{Da} = 0.1 \text{ s}^{-1}$

Psychrometric ratio,  $h_G/(h_D \rho_A s) = b = 2.34$

Heat transfer coefficients,  $h_L = 3h_G$

Latent heat of vaporisation of n-butanol,  $\lambda = 590 \text{ kJ/kg}$

Specific heat of liquid n-butanol,  $C_{Ll} = 2.5 \text{ kJ/kg K}$

Humid heat of gas,  $s = 1.05 \text{ kJ/kg K}$

Temperature (K)	Vapour pressure of butanol (kN/m <sup>2</sup> )
295	0.59
300	0.86
305	1.27
310	1.75
315	2.48
320	3.32
325	4.49
330	5.99
335	7.89
340	10.36
345	14.97
350	17.50

**Solution**

The first stage is to calculate the enthalpy of the saturated gas by way of the saturated humidity,  $\mathcal{H}_0$ , given by:

$$\mathcal{H}_0 = \frac{P_{w0}}{P - P_{w0}} \frac{M_w}{M_A} = \frac{P_{w0}}{(101.3 - P_{w0})} \left( \frac{74}{29} \right)$$

The enthalpy is then:

$$H_f = \frac{1}{(1 + \mathcal{H}_0)} \times 1.001(\theta_f - 273) + \mathcal{H}_0[2.5(\theta_f - 273) + 590] \text{ kJ/kg}$$

where 1.001 kJ/kg K is the specific heat of dry air.

Thus: 
$$H_f = \frac{1.001\theta_f - 273.27}{(1 + \mathcal{H}_0)} + \mathcal{H}_0(2.5\theta_f - 92.5) \text{ kJ/kg moist air}$$

The results of this calculation are presented in the following table and  $H_f$  is plotted against  $\theta_f$  in Figure 13.21.

The modified enthalpy at saturation  $H'_f$  is given by:

$$H'_f = \frac{(1.001\theta_f - 273.27)}{(1 + \mathcal{H}_0)} + \mathcal{H}_0[2.5(\theta_f - 273) + \lambda']$$

where from equation 13.70:  $\lambda' = \lambda/b = (590/2.34)$  or 252 kJ/kg

$$H'_f = \frac{(1.001\theta_f - 273.27)}{(1 + \mathcal{H}_0)} + \mathcal{H}_0(2.5\theta_f - 430.5) \text{ kJ/kg moist air}$$

These results are also given in the following Table and plotted as  $H'_f$  against  $\theta_f$  in Figure 13.21.

$\theta_f$ (K)	$P_{w0}$ (kN/m <sup>2</sup> )	$\mathcal{H}_0$ (kg/kg)	$(1.001\theta_f - 273.27)/(1 + \mathcal{H}_0)$ (kJ/kg)	$\mathcal{H}_0(2.5\theta_f - 92.5)$ (kJ/kg)	$H_f$ (kJ/kg)	$\mathcal{H}_0(2.5\theta_f - 430.5)$ (kJ/kg)	$H'_f$ (kJ/kg)
295	0.59	0.0149	21.70	9.61	31.31	4.57	26.28
300	0.86	0.0218	24.45	14.33	40.78	6.97	33.42
305	1.27	0.0324	31.03	21.71	52.74	10.76	41.79
310	1.75	0.0448	35.45	30.58	66.03	15.43	50.88
315	2.48	0.0640	39.52	44.48	84.00	22.85	62.37
320	3.32	0.0864	43.31	61.13	104.44	31.92	75.23
325	4.49	0.1183	46.55	85.18	131.73	45.19	91.74
330	5.99	0.1603	49.18	117.42	166.60	63.23	112.41
335	7.89	0.2154	51.07	160.47	211.54	87.67	136.73
340	10.36	0.2905	51.97	220.05	272.02	121.87	173.83
345	14.97	0.4422	49.98	340.49	390.47	191.03	241.01
350	17.50	0.5325	50.30	416.68	466.98	236.70	287.00

The bottom of the operating line (point  $a$ ) has coordinates,  $\theta_{L1} = 295$  K and  $H_{G1}$ , where:

$$H_{G1} = 1.05(290 - 273) = 17.9 \text{ kJ/kg.}$$

At a mean temperature of, say, 310 K, the density of air is:

$$\left( \frac{29}{22.4} \right) \left( \frac{273}{310} \right) = 1.140 \text{ kg/m}^3$$

and:

$$G' = (0.70 \times 1.140) = 0.798 \text{ kg/m}^2 \text{ s}$$

Thus, the slope of the operating line becomes:

$$\frac{L'C_L}{G'} = \frac{(0.25 \times 2.5)}{0.798} = 0.783 \text{ kJ/kg K}$$

and this is drawn in as  $AB$  in Figure 13.22 and at  $\theta_{L2} = 330$  K,  $H_{G2} = 46$  kJ/kg.

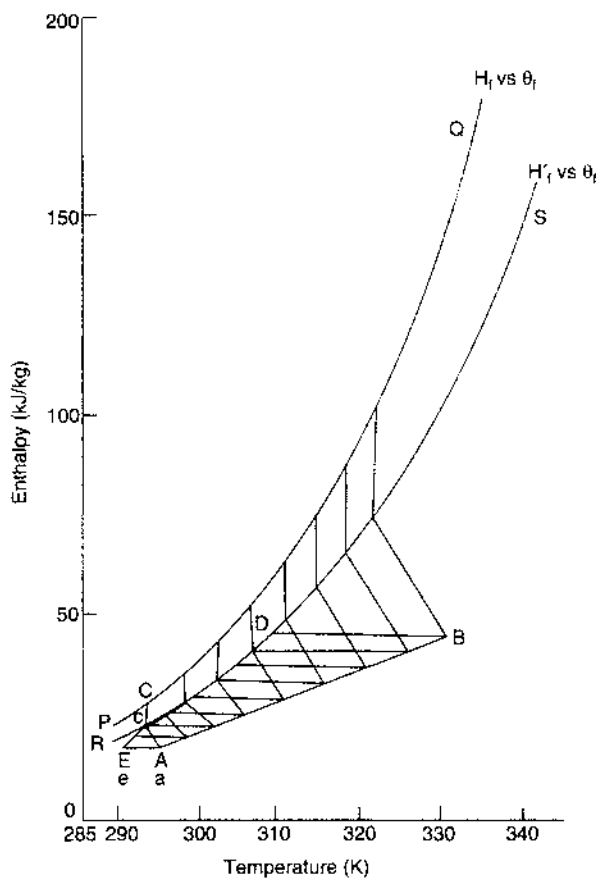


Figure 13.22. Graphical construction for Example 13.10

From equation 13.77:  $H'_G = [H_G + (b - 1)s(\theta_G - \theta_0)]/b$

$$H'_{G1} = \frac{17.9 + (2.34 - 1)1.05(290 - 273)}{2.34} = 17.87 \text{ kJ/kg}$$

Point *a* coincides with the bottom of the column.

$$\begin{aligned} \text{A line is drawn through } a \text{ of slope } -\frac{h_L}{h_D \rho b} &= -\left(\frac{3h_G}{h_D \rho b}\right) \cdot \left(\frac{h_D \rho s}{h_G}\right) \\ &= -3s = -3.15 \text{ kJ/kg K} \end{aligned}$$

This line meets curve *RS* at *c* ( $\theta_{f1}$ ,  $H'_{f1}$ ) to give the interface conditions at the bottom of the column. The corresponding air enthalpy is given by point *C* whose co-ordinates are:

$$\theta_{f1} = 293 \text{ K} \quad H_{f1} = 29.0 \text{ kJ/kg}$$

The difference between the ordinates of *c* and *a* gives the driving force in terms of the modified enthalpy at the bottom of the column, or:

$$(H'_{f1} - H'_{G1}) = (23.9 - 17.9) = 6.0 \text{ kJ/kg}$$

A similar construction is made at other points along the operating line with the results shown in the following table.

$\theta_i$ (K)	$H_G$ (kJ/kg)	$H'_G$ (kJ/kg)	$H'_f$ (kJ/kg)	$(H'_f - H'_G)$ (kJ/kg)	$1/(H'_f - H'_G)$ (kg/kJ)	Mean value in interval	Interval	Value of integral over interval
295	17.9	17.9	23.9	6.0	0.167			
300	22.0	22.0	29.0	7.0	0.143	0.155	4.1	0.636
305	26.0	26.0	35.3	9.3	0.108	0.126	4.0	0.504
310	30.0	30.0	42.1	12.1	0.083	0.096	4.0	0.384
315	34.0	34.0	50.0	16.0	0.063	0.073	4.0	0.292
320	38.1	38.1	57.9	19.8	0.051	0.057	4.1	0.234
325	42.0	42.0	66.7	24.7	0.041	0.046	3.9	0.179
330	46.0	46.0	75.8	29.8	0.034	0.0375	4.0	0.150
Value of integral = 2.379								

from which:

$$\int_{H_{G1}}^{H_{G2}} \frac{dH_G}{H'_f - H'_G} = 2.379$$

Substituting in equation 13.78:

$$\frac{bh_D \rho a z}{G'} = 2.379$$

and:

$$z = \frac{(2.379 \times 0.798)}{(2.34 \times 0.1)} = \underline{\underline{8.1 \text{ m}}}$$

It remains to evaluate the change in gas conditions.

Point  $e$ , ( $\theta_{G1} = 290 \text{ K}$ ,  $H_{G1} = 17.9 \text{ kJ/kg}$ ) represents the condition of the inlet gas.  $ec$  is now drawn in, and from equation 13.75, this represents  $dH_G/d\theta_G$ . As for the air-water system, this construction is continued until the gas enthalpy reaches  $H_{G2}$ . The final point is given by  $D$  at which  $\theta_{G2} = 308 \text{ K}$ .

It is fortuitous that, in this problem,  $H'_G = H_G$ . This is not always the case and reference should be made to Section 13.7 for elaboration of this point.

## 13.8. FURTHER READING

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## 13.10. NOMENCLATURE

		Units in SI system	Dimensions in M, N, L, T, $\theta$
$A$	Interfacial area	$m^2$	$L^2$
$A_h$	Base area of hyperbolic tower	$m^2$	$L^2$
$a$	Interfacial area per unit volume of column	$m^2/m^3$	$L^{-1}$
$h$	Psychrometric ratio ( $h/h_D \rho_{AS}$ )	—	—
$C_g$	Specific heat of gas at constant pressure	$J/kg \text{ } ^\circ K$	$L^2 T^{-2} \theta^{-1}$
$C_l$	Specific heat of liquid	$J/kg \text{ } ^\circ K$	$L^2 T^{-2} \theta^{-1}$
$C_p$	Specific heat of gas and vapour mixture at constant pressure	$J/kg \text{ } ^\circ K$	$L^2 T^{-2} \theta^{-1}$
$C_s$	Specific heat of solid	$J/kg \text{ } ^\circ K$	$L^2 T^{-2} \theta^{-1}$
$C_r$	Performance coefficient or efficiency factor	—	—
$C_v$	Specific heat of vapour at constant pressure	$J/kg \text{ } ^\circ K$	$L^2 T^{-2} \theta^{-1}$
$c$	Mass concentration of vapour	$kg/m^3$	$ML^{-3}$
$c_s$	Mass concentration of vapour in saturated gas	$kg/m^3$	$ML^{-3}$
$D_t$	Duty coefficient of tower (equation 13.31)	—	—
$f$	Correction factor for mean driving force	—	—
$G$	Mass rate of flow of gas per unit area	$kg/m^2 \text{ } s$	$ML^{-2} T^{-1}$
$H$	Enthalpy of humid gas per unit mass of dry gas	$J/kg$	$L^2 T^{-2}$
$H_a$	Enthalpy per unit mass, of dry gas	$J/kg$	$L^2 T^{-2}$
$H_u$	Enthalpy per unit mass, of vapour	$J/kg$	$L^2 T^{-2}$
$H_1$	Enthalpy of stream of gas, per unit mass of dry gas	$J/kg$	$L^2 T^{-2}$
$H_2$	Enthalpy of another stream of gas, per unit mass of dry gas	$J/kg$	$L^2 T^{-2}$
$H_3$	Enthalpy per unit mass of liquid or vapour	$J/kg$	$L^2 T^{-2}$
$H'$	Modified enthalpy of humid gas defined by (13.69)	$J/kg$	$L^2 T^{-2}$
$\Delta H$	Enthalpy driving force ( $H_f - H_G$ )	$J/kg$	$L^2 T^{-2}$
$\Delta H'$	Change in air enthalpy on passing through tower	$J/kg$	$L^2 T^{-2}$
$h$	Heat transfer coefficient	$W/m^2 \text{ } ^\circ K$	$MT^{-3} \theta^{-1}$
$h_D$	Mass transfer coefficient	$kmol/(kmol/m^3)m^2 \text{ } s$	$LT^{-1}$
$h_G$	Heat transfer coefficient for gas phase	$W/m^2 \text{ } ^\circ K$	$MT^{-3} \theta^{-1}$
$h_L$	Heat transfer coefficient for liquid phase	$W/m^2 \text{ } ^\circ K$	$MT^{-3} \theta^{-1}$
$\mathcal{H}$	Humidity	$kg/kg$	—
$\mathcal{H}_s$	Humidity of gas saturated at the adiabatic saturation temperature	$kg/kg$	—
$\mathcal{H}_w$	Humidity of gas saturated at the wet-bulb temperature	$kg/kg$	—
$\mathcal{H}_0$	Humidity of saturated gas	$kg/kg$	—
$\mathcal{H}_1$	Humidity of a gas stream	$kg/kg$	—
$\mathcal{H}_2$	Humidity of second gas stream	$kg/kg$	—
$L'$	Mass rate of flow of liquid per unit area	$kg/m^2 \text{ } s$	$ML^{-2} T^{-1}$
$M_A$	Molecular weight of gas	$kg/kmol$	$MN^{-1}$
$M_u$	Molecular weight of vapour	$kg/kmol$	$MN^{-1}$
$m, m_1, m_2$	Masses of dry gas	$kg$	$M$
$m_3$	Mass of liquid or vapour	$kg$	$M$
$P$	Total pressure	$N/m^2$	$ML^{-1} T^{-2}$
$P_1$	Mean partial pressure of gas	$N/m^2$	$ML^{-1} T^{-2}$
$P_u$	Partial pressure of vapour	$N/m^2$	$ML^{-1} T^{-2}$
$P_{u,s}$	Partial pressure of vapour in saturated gas	$N/m^2$	$ML^{-1} T^{-2}$
$\dot{Q}$	Rate of transfer of heat to liquid surface	$W$	$ML^2 T^{-3}$
$R$	Universal gas constant	$8314 \text{ } J/kmol \text{ } ^\circ K$	$MN^{-1} L^2 T^{-2} \theta^{-1}$
$s$	Humid heat of gas	$J/kg \text{ } ^\circ K$	$L^2 T^{-2} \theta^{-1}$
$T$	Absolute temperature	$K$	$\theta$

		Units in SI system	Dimensions in M, N, L, T, $\theta$
$\Delta T$	Change in water temperature in passing through the tower	K	$\theta$
$\Delta T'$	(Temperature of air leaving packing — ambient dry-bulb temperature)	K	$\theta$
$V$	Active volume per plan area of column	$\text{m}^3/\text{m}^2$	<b>L</b>
$W_L$	Water loading on tower	$\text{kg/s}$	<b>MT<sup>-1</sup></b>
$w$	Rate of evaporation	$\text{kg/s}$	<b>MT<sup>-1</sup></b>
$Z$	Percentage humidity	—	—
$z$	Height from bottom of tower	m	<b>L</b>
$z_i$	Height of cooling tower	m	<b>L</b>
$\theta$	Temperature of gas stream	K	$\theta$
$\theta_0$	Reference temperature, taken as the melting point of the material	K	$\theta$
$\theta_s$	Adiabatic saturation temperature	K	$\theta$
$\theta_w$	Wet bulb temperature	K	$\theta$
$\lambda$	Latent heat of vaporisation per unit mass, at datum temperature	J/kg	<b>L<sup>2</sup> T<sup>-2</sup></b>
$\lambda_i$	Latent heat of freezing per unit mass, at datum temperature	J/kg	<b>L<sup>2</sup> T<sup>-2</sup></b>
$\lambda$	Modified latent heat of vaporisation per unit mass defined by (13.68)	J/kg	<b>L<sup>2</sup> T<sup>-2</sup></b>
$\rho$	Mean density of gas and vapour	$\text{kg/m}^3$	<b>ML<sup>-3</sup></b>
$\rho_A$	Mean density of gas at partial pressure $P_A$	$\text{kg/m}^3$	<b>ML<sup>-3</sup></b>
$Pr$	Prandtl number	—	—
$Sc$	Schmidt number	—	—

Suffixes 1, 2, *f*, *L*, *G* denote conditions at the bottom of the tower, the top of the tower, the interface, the liquid, and the gas, respectively.

Suffix *m* refers to the mean water temperature.