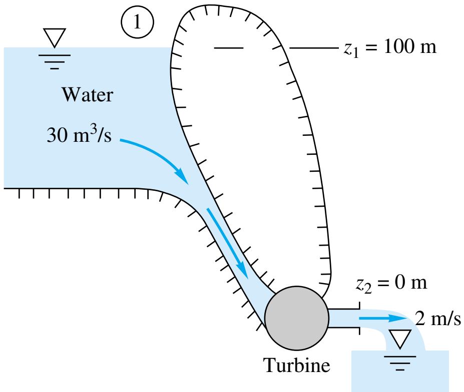
EXAMPLE 3.19

A hydroelectric power plant (Fig. E3.19) takes in 30 m^3/s of water through its turbine and discharges it to the atmosphere at $V_2 = 2$ m/s. The head loss in the turbine and penstock system is $h_f = 20$ m. Assuming turbulent flow, $\alpha \approx 1.06$, estimate the power in MW extracted by the turbine.



E3.19

Solution

We neglect viscous work and heat transfer and take section 1 at the reservoir surface (Fig. E3.19), where $V_1 \approx 0$, $p_1 = p_{atm}$, and $z_1 = 100$ m. Section 2 is at the turbine outlet. The steady-flow energy equation (3.71) becomes, in head form,

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_t + h_f$$

$$\frac{p_a}{\gamma} + \frac{1.06(0)^2}{2(9.81)} + 100 \text{ m} = \frac{p_a}{\gamma} + \frac{1.06(2.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} + 0 \text{ m} + h_t + 20 \text{ m}$$

The pressure terms cancel, and we may solve for the turbine head (which is positive):

$$h_t = 100 - 20 - 0.2 \approx 79.8 \text{ m}$$

The turbine extracts about 79.8 percent of the 100-m head available from the dam. The total power extracted may be evaluated from the water mass flow:

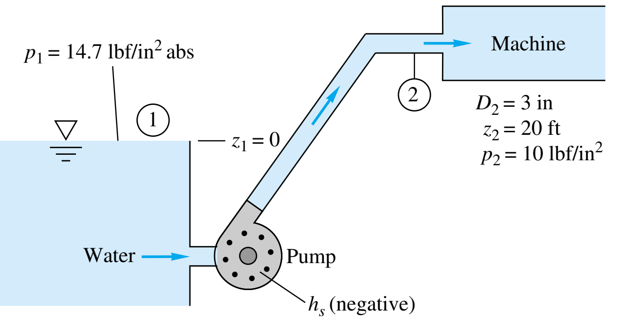
$$P = \dot{m} w_s = (\rho Q)(g h_t) = (998 \text{ kg/m}^3)(30 \text{ m}^3\text{/s})(9.81 \text{ m/s}^2)(79.8 \text{ m})$$

$$= 23.4 \text{ E6 kg} \cdot \text{m}^2/\text{s}^3 = 23.4 \text{ E6 N} \cdot \text{m/s} = 23.4 \text{ MW} \qquad \textit{Ans.}$$

The turbine drives an electric generator which probably has losses of about 15 percent, so the net power generated by this hydroelectric plant is about 20 MW.

EXAMPLE 3.20

The pump in Fig. E3.20 delivers water (62.4 lbf/ft^3) at 3 ft^3/s to a machine at section 2, which is 20 ft higher than the reservoir surface. The losses between 1 and 2 are given by $h_f = KV_2^2/(2g)$,



E3.20

where $K \approx 7.5$ is a dimensionless loss coefficient (see Sec. 6.7). Take $\alpha \approx 1.07$. Find the horsepower required for the pump if it is 80 percent efficient.

Solution

If the reservoir is large, the flow is steady, with $V_1 \approx 0$. We can compute V_2 from the given flow rate and the pipe diameter:

$$V_2 = \frac{Q}{A_2} = \frac{3\ \text{ft}^3/\text{s}}{(\pi/4)(\frac{3}{12}\ \text{ft})^2} = 61.1\ \text{ft/s}$$

The viscous work is zero because of the solid walls and near-one-dimensional inlet and exit. The steady-flow energy equation (3.71) becomes

$$\frac{p_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{\alpha_2 V_2^2}{2g} + z_2 + h_s + h_f$$

Introducing $V_1 \approx 0$, $z_1 = 0$, and $h_f = KV_2^2/(2g)$, we may solve for the pump head:

$$h_s = \frac{p_1 - p_2}{\gamma} - z_2 - (\alpha_2 + K)\left(\frac{V_2^2}{2g}\right)$$

The pressures should be in lbf/ft^2 for consistent units. For the given data, we obtain

$$h_s = \frac{(14.7 - 10.0)(144)\ \text{lbf/ft}^2}{62.4\ \text{lbf/ft}^3} - 20\ \text{ft} - (1.07 + 7.5)\,\frac{(61.1\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)}$$

$$= 11 - 20 - 497 = -506\ \text{ft}$$

The pump head is negative, indicating work done *on* the fluid. As in Example 3.19, the power delivered is computed from

$$P = \dot{m} w_s = \rho Q g h_s = (1.94\ \text{slug/ft}^3)(3.0\ \text{ft}^3/\text{s})(32.2\ \text{ft/s}^2)(-507\ \text{ft}) = -94{,}900\ \text{ft} \cdot \text{lbf/s}$$

or

$$\text{hp} = \frac{94{,}900\ \text{ft} \cdot \text{lbf/s}}{550\ \text{ft} \cdot \text{lbf/(s} \cdot \text{hp)}} \approx 173\ \text{hp}$$

We drop the negative sign when merely referring to the "power" required. If the pump is 80 percent efficient, the input power required to drive it is

$$P_{\text{input}} = \frac{P}{\text{efficiency}} = \frac{173 \text{ hp}}{0.8} \approx 216 \text{ hp} \qquad \textit{Ans.}$$

The inclusion of the kinetic-energy correction factor α in this case made a difference of about 1 percent in the result.

3.7 Frictionless Flow: The Bernoulli Equation

Closely related to the steady-flow energy equation is a relation between pressure, velocity, and elevation in a frictionless flow, now called the *Bernoulli equation*. It was stated (vaguely) in words in 1738 in a textbook by Daniel Bernoulli. A complete derivation of the equation was given in 1755 by Leonhard Euler. The Bernoulli equation is very famous and very widely used, but one should be wary of its restrictions—all fluids are viscous and thus all flows have friction to some extent. To use the Bernoulli equation correctly, one must confine it to regions of the flow which are nearly frictionless. This section (and, in more detail, Chap. 8) will address the proper use of the Bernoulli relation.

Consider Fig. 3.15, which is an elemental fixed streamtube control volume of variable area $A(s)$ and length ds, where s is the streamline direction. The properties (ρ, V, p) may vary with s and time but are assumed to be uniform over the cross section A. The streamtube orientation θ is arbitrary, with an elevation change $dz = ds \sin \theta$. Friction on the streamtube walls is shown and then neglected—a very restrictive assumption.

Conservation of mass (3.20) for this elemental control volume yields

$$\frac{d}{dt}\left(\int_{\text{CV}} \rho \, d\mathcal{V}\right) + \dot{m}_{\text{out}} - \dot{m}_{\text{in}} = 0 \approx \frac{\partial \rho}{\partial t} d\mathcal{V} + d\dot{m}$$

where $\dot{m} = \rho A V$ and $d\mathcal{V} \approx A \, ds$. Then our desired form of mass conservation is

$$d\dot{m} = d(\rho A V) = -\frac{\partial \rho}{\partial t} A \, ds \qquad (3.74)$$

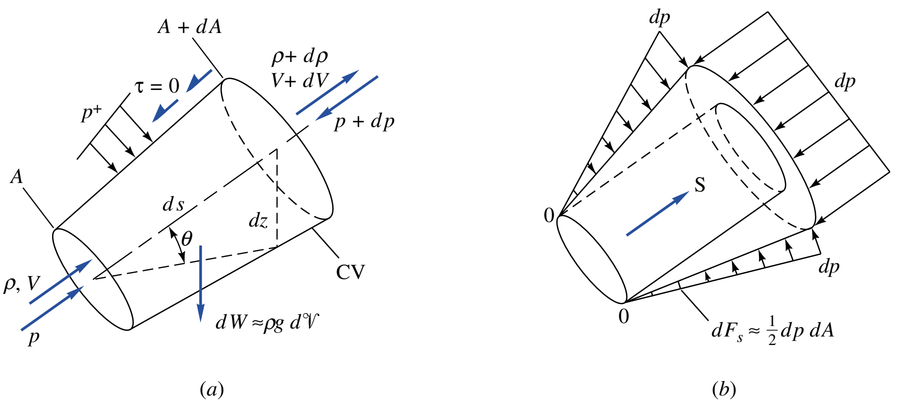


Fig. 3.15 The Bernoulli equation for frictionless flow along a streamline: (*a*) forces and fluxes; (*b*) net pressure force after uniform subtraction of p.