The area element in Eq. (2.6.3) should be $\delta x\ \delta y$, and not the strip shown in Fig. 2.22.

After solving for the coordinates of pressure center,

$$x_p = \frac{1}{F}\int_A xp\,dA \tag{2.6.5}$$

$$y_p = \frac{1}{F}\int_A yp\,dA \tag{2.6.6}$$

In many applications Eqs. (2.6.5) and (2.6.6) may be evaluated most conveniently through graphical integration; for simple areas they may be transformed into general formulas as follows:[1]

$$x_p = \frac{1}{\gamma\bar{y}A\sin\theta}\int_A x\gamma y\sin\theta\,dA = \frac{1}{\bar{y}A}\int_A xy\,dA = \frac{I_{xy}}{\bar{y}A} \tag{2.6.7}$$

In Eqs. (A.10), of Appendix A, and (2.6.7),

$$x_p = \frac{\bar{I}_{xy}}{\bar{y}A} + \bar{x} \tag{2.6.8}$$

When either of the centroidal axes, $x = \bar{x}$ or $y = \bar{y}$, is an axis of symmetry for the surface, $\bar{I}_{xy}$ vanishes and the pressure center lies on $x = \bar{x}$. Since $\bar{I}_{xy}$ may be either positive or negative, the pressure center may lie on either side of the line $x = \bar{x}$. To determine y_p by formula, with Eqs. (2.6.2) and (2.6.6),

$$y_p = \frac{1}{\gamma\bar{y}A\sin\theta}\int_A y\gamma y\sin\theta\,dA = \frac{1}{\bar{y}A}\int_A y^2\,dA = \frac{I_x}{\bar{y}A} \tag{2.6.9}$$

In the parallel-axis theorem for moments of inertia

$$I_x = I_G + \bar{y}^2A$$

If I_x is eliminated from Eq. (2.6.9)

$$y_p = \frac{I_G}{\bar{y}A} + \bar{y} \tag{2.6.10}$$

or

$$y_p - \bar{y} = \frac{I_G}{\bar{y}A} \tag{2.6.11}$$

I_G is always positive; hence, $y_p - \bar{y}$ is always positive, and the pressure center is always below the centroid of the surface. It should be emphasized that $\bar{y}$ and $y_p - \bar{y}$ are distances in the plane of the surface.

Example 2.11: The triangular gate CDE (Fig. 2.23) is hinged along CD and is opened by a normal force P applied at E. It holds oil, sp gr 0.80, above it and

[1] See Appendix A.

is open to the atmosphere on its lower side. Neglecting the weight of the gate determine (*a*) the magnitude of force exerted on the gate, by integration and by Eq. (2.6.2); (*b*) the location of pressure center; (*c*) the force P necessary to open the gate.

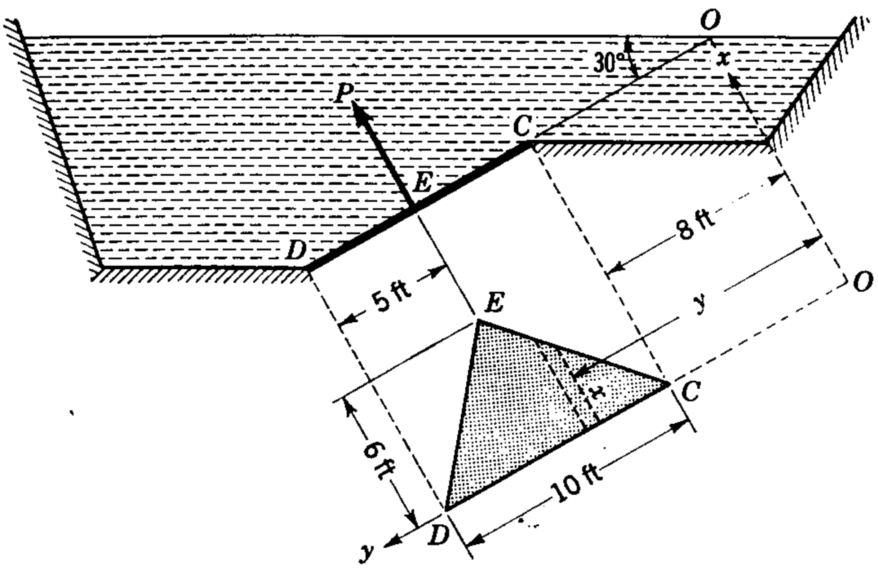

FIG. 2.23. Triangular gate.

a. By integration with reference to Fig. 2.23

$$F = \int_A p\,dA = \gamma \sin\theta \int yx\,dy = \gamma \sin\theta \int_8^{13} xy\,dy + \gamma \sin\theta \int_{13}^{18} xy\,dy$$

When $y = 8, x = 0$, and when $y = 13, x = 6$, with x varying linearly with y, thus

$$x = ay + b \qquad 0 = 8a + b \qquad 6 = 13a + b$$

in which the coordinates have been substituted to find x in terms of y. After solving for a and b,

$$a = \tfrac{6}{5} \qquad b = -\tfrac{48}{5}, \qquad x = \tfrac{6}{5}(y - 8)$$

Similarly $y = 13, x = 6; y = 18, x = 0$; and $x = \frac{6}{5}(18 - y)$. Hence

$$F = \gamma \sin\theta \frac{6}{5}\left[\int_8^{13} (y - 8)y\,dy + \int_{13}^{18} (18 - y)y\,dy\right]$$

After integrating and substituting for $\gamma \sin\theta$,

$$F = 62.4 \times 0.8 \times 0.50 \times \frac{6}{5}\left[\left(\frac{y^3}{3} - 4y^2\right)_8^{13} + \left(9y^2 - \frac{y^3}{3}\right)_{13}^{18}\right] = 9734.4 \text{ lb}$$

By Eq. (2.6.2)

$$F \doteq p_G A = \gamma \bar{y} \sin\theta\, A = 62.4 \times 0.80 \times 0.50 \times 30 \times 13 = 9734.4 \text{ lb}$$

b. With the axes as shown, $\bar{x} = 2.0$, $\bar{y} = 13$. In Eq. (2.6.8)

$$x_p = \frac{\bar{I}_{xy}}{\bar{y}A} + \bar{x}$$

$\bar{I}_{xy}$ is zero owing to symmetry about the centroidal axis parallel to the x-axis; hence $\bar{x} = x_p = 2.0$ ft. In Eq. (2.6.11),

$$y_p - \bar{y} = \frac{I_G}{\bar{y}A} = 2 \times \frac{1 \times 6 \times 5^3}{12 \times 13 \times 30} = 0.32 \text{ ft}$$

i.e., the pressure center is 0.32 ft below the centroid, measured in the plane of the area.

c. When moments about CD are taken and the action of the oil is replaced by the resultant,

$$P \times 6 = 9734.4 \times 2 \qquad P = 3244.8 \text{ lb}$$

The Pressure Prism. The concept of the pressure prism provides another means for determining the magnitude and location of the resultant force on an inclined plane surface. The volume of the pressure prism is the magnitude of the force and the resultant force passes through the centroid of the prism. The surface is taken as the base of the prism, and its altitude at each point is determined by the pressure γh laid off to an appropriate scale (Fig. 2.24). Since the pressure increases linearly with distance from the free surface, the upper surface of the prism is in a plane with its trace OM shown in Fig. 2.24. The force acting on an elemental area δA is

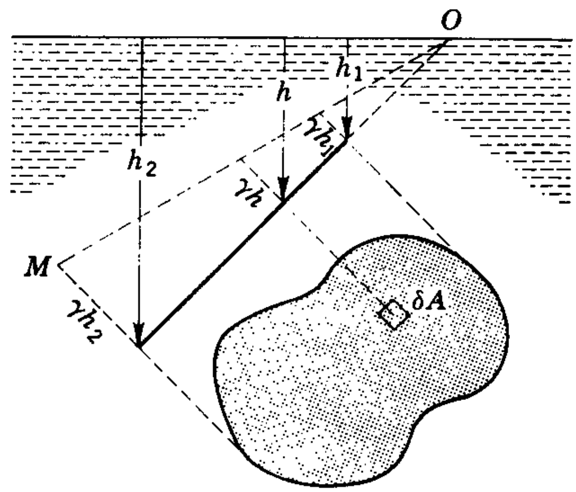

FIG. 2.24. Illustration of pressure prism.

$$\delta F = \gamma h \, \delta A = \delta \mathcal{V} \qquad (2.6.12)$$

which is an element of volume of the pressure prism. After integrating, $F = \mathcal{V}$, the volume of the pressure prism equals the magnitude of the resultant force acting on one side of the surface.

Equations (2.6.5) and (2.6.6),

$$x_p = \frac{1}{\mathcal{V}} \int_{\mathcal{V}} x \, d\mathcal{V} \qquad y_p = \frac{1}{\mathcal{V}} \int_{\mathcal{V}} y \, d\mathcal{V} \qquad (2.6.13)$$

show that x_p, y_p are distances to the *centroid of the pressure* prism.[1] Hence, the line of action of the resultant passes through the centroid of the pressure prism. For some simple areas the pressure prism is more convenient than either integration or formula. For example, a rectangular area with one edge in the free surface has a wedge-shaped prism. Its centroid is one-third the altitude from the base; hence, the pressure center is one-third the altitude from its lower edge.

[1] Appendix A, Eq. (A.5).