

(a) For the 1st stage: $AX_f + SY_2 = AX_1 + SY_1$

(b) For the n th stage: $AX_{n-1} + SY_{n+1} = AX_n + SY_n$

(c) For the whole unit: $AX_f + SY_{n+1} = AX_n + SY_1$

or:
$$Y_{n+1} = \frac{A}{S}(X_n - X_f) + Y_1 \quad (13.8)$$

This is the equation of a straight line of slope A/S , known as the *operating line*, which passes through the points (X_f, Y_1) and (X_n, Y_{n+1}) . In Figure 13.14, the equilibrium relation, Y_n against X_n , and the operating line are drawn in, and the number of stages required to pass from X_f to X_n is found by drawing in steps between the operating line and the equilibrium curve. In this example, four stages are required, and (X_n, Y_{n+1}) corresponds to (X_4, Y_5) . It may be noted that the operating line connects the compositions of the raffinate stream leaving and the fresh solvent stream entering a unit, X_n and Y_{n+1} , respectively.

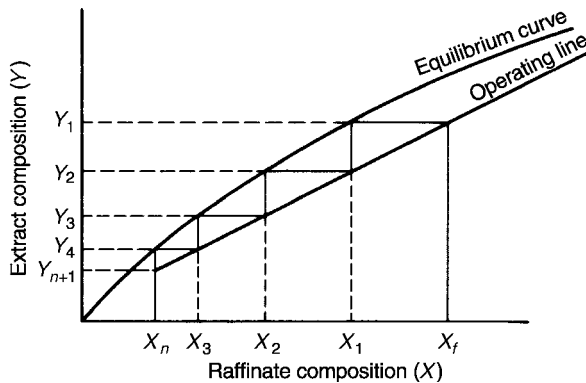


Figure 13.14. Graphical method for determining the number of stages for the process shown in Figure 13.13, using immiscible solvents

Example 13.1

160 cm³/s of a solvent **S** is used to treat 400cm³/s of a 10 per cent by mass solution of **A** in **B**, in a three-stage countercurrent multiple-contact liquid–liquid extraction plant. What is the composition of the final raffinate?

Using the same total amount of solvent, evenly distributed between the three stages, what would be the composition of the final raffinate if the equipment were used in a simple multiple-contact arrangement?

Equilibrium data:

kg A /kg B :	0.05	0.10	0.15
kg A /kg S :	0.069	0.159	0.258
Densities (kg/m ³):	$\rho_A = 1200$,	$\rho_B = 1000$,	$\rho_S = 800$

Solution(a) *Countercurrent operation*Considering the solvent **S**, $160\text{cm}^3/\text{s} = 1.6 \times 10^{-4}\text{m}^3/\text{s}$ and: mass flowrate = $(1.6 \times 10^{-4} \times 800) = 0.128\text{ kg/s}$ Considering the solution, $400\text{cm}^3/\text{s} = 4 \times 10^{-4}\text{ m}^3/\text{s}$
containing, say, $a\text{ m}^3/\text{s}$ **A** and $(5 \times 10^{-4} - a)\text{ m}^3/\text{s}$ **B**.Thus: mass flowrate of **A** = $1200a\text{ kg/s}$ and: mass flowrate of **B** = $(4 \times 10^{-4} - a)1000 = (0.4 - 1000a)\text{ kg/s}$ a total of: $(0.4 + 200a)\text{ kg/s}$

The concentration of the solution is:

$$0.10 = 1200a / (0.4 + 200a)$$

Thus: $a = 3.39 \times 10^{-5}\text{ m}^3/\text{s}$ mass flowrate of **A** = 0.041 kg/s , mass flowrate of **B** = 0.366 kg/s and: ratio of **A/B** in the feed, $X_f = (0.041/0.366) = 0.112\text{ kg/kg}$ The equilibrium data are plotted in Figure 13.15 and the value of $X_f = 0.112\text{ kg/kg}$ is marked in. The slope of the equilibrium line is:

$$(\text{mass flowrate of B}) / (\text{mass flowrate of S}) = (0.366/0.128) = 2.86$$

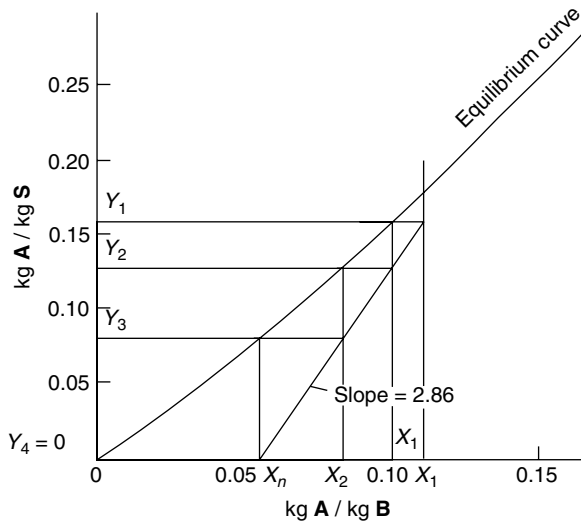


Figure 13.15. Construction for Example 13.1

Since pure solvent is added, $Y_{n+1} = Y_4 = 0$ and a line of slope 2.86 is drawn in such that stepping off from $X_f = 0.112$ kg/kg to $Y_4 = 0$ gives exactly three stages.

When $Y_4 = 0$, $X_n = X_3 = 0.057$ kg/kg,

Thus: the composition of final raffinate is 0.057 kg A/kg B

(b) *Multiple contact*

In this case, $(0.128/3) = 0.0427$ kg/s of pure solvent **S** is fed to each stage.

Stage 1

$$X_f = (0.041/0.366) = 0.112 \text{ kg/kg}$$

and from the equilibrium curve, the extract contains 0.18 A/kg **S** and $(0.18 \times 0.0427) = 0.0077$ kg/s **A**.

Thus: raffinate from stage 1 contains $(0.041 - 0.0077) = 0.0333$ kg/s **A** and 0.366 kg/s **B**

and: $X_1 = (0.0333/0.366) = 0.091$ kg/kg

Stage 2

$$X_1 = 0.091 \text{ kg/kg}$$

and from Figure 13.15 the extract contains 0.14 kg A/kg **S**

or: $(0.14 \times 0.0427) = 0.0060$ kg/s **A**

Thus: the raffinate from stage 2 contains $(0.0333 - 0.0060) = 0.0273$ kg/s **A** and 0.366 kg/s **B**

Thus: $X_2 = (0.0273/0.366) = 0.075$ kg/kg

Stage 3

$$X_2 = 0.075 \text{ kg/kg}$$

and from Figure 13.15, the extract contains 0.114 kg A/kg **S**

or: $(0.114 \times 0.0427) = 0.0049$ kg/s **A**.

Thus: the raffinate from stage 3 contains $(0.0273 - 0.0049) = 0.0224$ kg/s **A** and 0.366 kg/s **B**

and: $X_3 = (0.0224/0.366) = 0.061$ kg/kg

Thus: the composition of final raffinate = 0.061 kg A/kg B

13.4.4. Countercurrent contact with partially miscible solvents

In this case the arrangement of the equipment is the same as for immiscible solvents although, as the amounts of solvent in the extract and raffinate streams are varying, the material balance is taken for the total streams entering and leaving each stage.

With the notation as shown in Figure 13.13, if the feed F , the final extract E_1 , the fresh solvent $S = \text{stream } E_{n+1}$ and, the final raffinate R_n are fixed, then making material balances:

(a) *Over the first unit*

$$F + E_2 = R_1 + E_1$$

and: $F - E_1 = R_1 - E_2 = P$, say — the difference stream (13.9)

(b) *Over stages 1 to n*

$$F + E_{n+1} = R_n + E_1 = M, \text{ say} \quad (13.10)$$

and: $F - E_1 = R_n - E_{n+1} = P$. (13.11)

(c) *Over the unit n*

$$R_{n-1} + E_{n+1} = E_n + R_n$$

and: $R_{n-1} - E_n = R_n - E_{n+1} = P$ (13.12)

Thus the difference in quantity between the raffinate leaving a stage R_n , and the extract entering from next stage E_{n+1} , is constant. Similarly, it can be shown that the difference between the amounts of each component in the raffinate and the extract streams is constant. This means that, with the notation of a triangular diagram, lines joining any two points representing R_n and E_{n+1} pass through a common pole. The number of stages required to so from an initial concentration F to a final raffinate concentration R_n may then be found using a triangular diagram, shown in Figure 13.16.

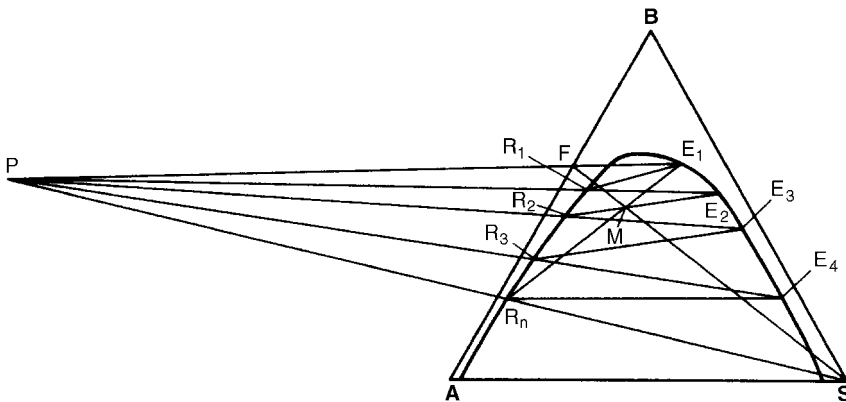


Figure 13.16. Graphical method for determining the number of stages for the process shown in Figure 13.13, using partially miscible solvents

If the points F and S representing the compositions of the feed and fresh solvent S are joined, then the composition of a mixture of F and S is shown by point M where:

$$\frac{MS}{MF} = \frac{\text{mass of } F}{\text{mass of } S}$$

A line is drawn from R_n through M to give E_1 on the binodal curve and E_1F and SR_n to meet at the pole P . It may be noted that P represents an imaginary mixture, as described for the leaching problems discussed in Chapter 10.

In an ideal stage, the extract E_1 leaves in equilibrium with the raffinate R_1 , so that the point R_1 is at the end of the tie line through E_1 . To determine the extract E_2 , PR_1 is drawn to cut the binodal curve at E_2 . The points R_2, E_3, R_3, E_4 , and so on, may be found in the same way. If the final tie line, say ER_4 , does not pass through R_n , then the amount of solvent added is incorrect for the desired change in composition. In general, this does not invalidate the method, since it gives the required number of ideal stages with sufficient accuracy.

Example 13.2

A 50 per cent solution of solute **C** in solvent **A** is extracted with a second solvent **B** in a counter-current multiple contact extraction unit. The mass of **B** is 25 per cent of that of the feed solution, and the equilibrium data are as given in Figure 13.17.

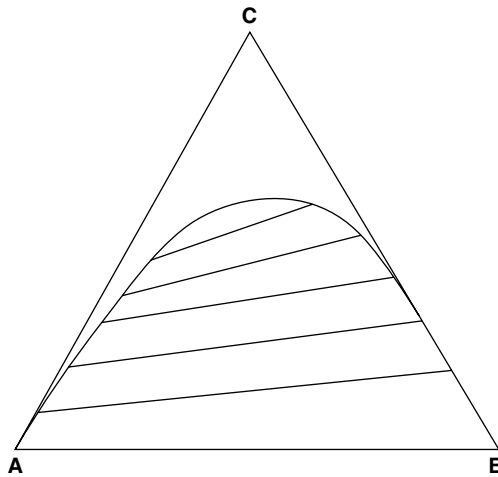


Figure 13.17. Equilibrium data for Example 13.2.

Determine the number of ideal stages required and the mass and concentration of the first extract if the final raffinate contains 15 per cent of solute **C**.

Solution

The equilibrium data are replotted in Figure 13.18 and F , representing the feed, is drawn in on AC at $C = 0.50, A = 0.50$. FB is joined and M located such that $FM/MB = 0.25$. R_n is located on the equilibrium curve such that $C = 0.15$. In fact $B = 0.01$ and $A = 0.84$. E_1 is located by projecting R_nM on to the curve and the pole P by projecting E_1F and BR_n . R_1 is found by projecting from

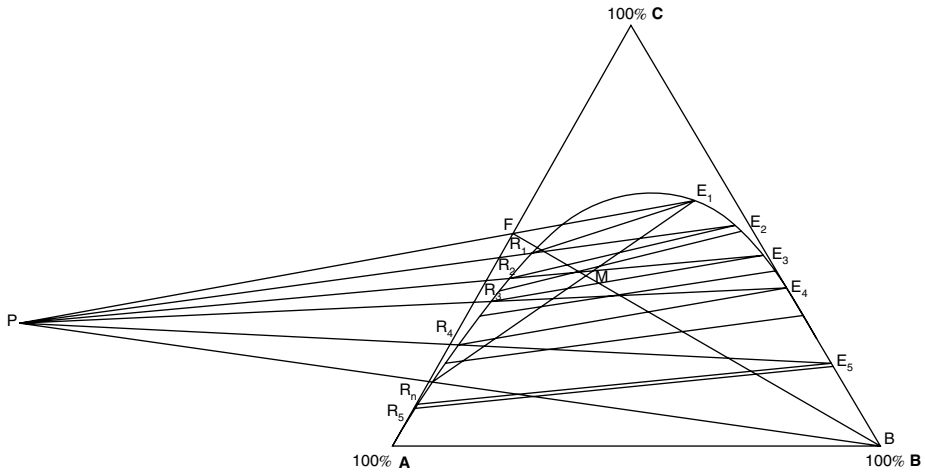


Figure 13.18. Graphical construction for Example 13.2.

E_1 along a tie-line and E_2 as the projection of PR_1 . The working is continued in this way and it is found that R_5 is below R_n and hence 5 ideal stages are required.

From Figure 13.18 the concentration of extract E_1 is 9 per cent A, 58 per cent C, and 33 per cent B.

13.4.5. Continuous extraction in columns

As SHERWOOD and PIGFORD⁽³⁾ point out, the use of spray towers, packed towers or mechanical columns enables continuous countercurrent extraction to be obtained in a similar manner to that in gas absorption or distillation. Applying the two-film theory of mass transfer, explained in detail in Volume 1, Chapter 10, the concentration gradients for transfer to a desired solute from a raffinate to an extract phase are as shown in Figure 13.19, which is similar to Figure 12.1 for gas absorption.

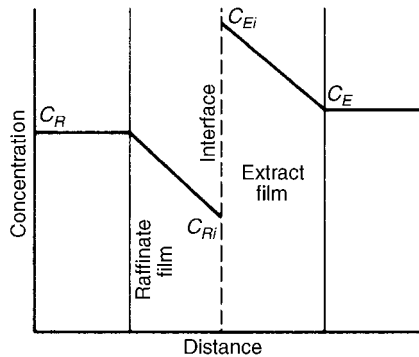


Figure 13.19. Concentration profile near an interface